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A METHOD OF PREDICTING
STATICAL STABILITY

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Cambridge 39, Mass.

Office of
G. C. Manning

Memo to: Capt. W. H. Buracker, USN.

From: Professor G. C. Manning

Date: 30 September 1946

Subj: Thesis work of LCDR John W. Crawford, USN
LCDR Robert C. Gooding, USN
LCDR Stuart C. Jones, USN
LCDR Archie P. Kelley, USN

The officers listed above have recently submitted a Master's Thesis done under my supervision entitled:

"A Method of Predicting Statical Stability."

The authors have made a thorough study to ascertain the practicability of deducing curves or equations from which a curve of statical stability can be predicted from the principal dimensions and coefficients of fineness and estimated height of center of gravity, analogous to Taylor's method for predicting a curve of E.H.P. from similar data. Such an accomplishment could have wide use in preliminary design. The authors have made an excellent beginning and are to be commended highly notwithstanding the fact that further studies will be required before this method can be put to practical use.

/s/ G. C. Manning

G. C. Manning
Professor of Naval Architecture

Cambridge, Massachusetts,
September 16, 1946.

Professor J. S. Newell,
Secretary of the Faculty,
Massachusetts Institute of Technology,
Cambridge, Massachusetts.

Dear Sir:

In accordance with the requirements for the Degree of Master of Science in Naval Construction and Engineering, we submit herewith a thesis entitled, "A Method for Predicting Statical Stability".

Respectfully,

A METHOD OF PREDICTING STATICAL STABILITY

by

Archie P. Kelley
Lieut. Commander, U. S. Navy
B.S., U. S. Naval Academy, 1941

Stuart C. Jones
Lieut. Commander, U. S. Navy
B.S., U. S. Naval Academy, 1941

John W. Crawford
Lieut. Commander, U. S. Navy
B.S., U. S. Naval Academy, 1941

and

Robert C. Gooding
Lieut. Commander, U. S. Navy
B.S., U. S. Naval Academy, 1941

Submitted in Partial Fulfillment

of the Requirements for the Degree of

Master of Science in Naval Construction and Engineering

from the

Massachusetts Institute of Technology

1946

ACKNOWLEDGMENT

The authors wish to express their appreciation to Professor G. C. Manning for his valuable advice and assistance, and for the suggestion which originally inspired this investigation.

TABLE OF CONTENTS

SYMBOLS

B - 1. beam; 2. center of buoyancy.

b - block coefficient.

BM - transverse metacentric radius.

D - depth.

G - center of gravity.

GM - transverse metacentric height.

GZ - righting arm.

H - draft.

I - transverse moment of inertia of waterplane.

K - keel.

KB - vertical distance from keel to center of buoyancy.

KG - vertical distance from keel to center of gravity.

L - 1. length of ship; 2. a parameter used in Taylor's Mathematical Lines.

l - longitudinal coefficient.

m - midship coefficient.

p - waterline coefficient.

θ - angle of inclination.

Additional symbols appearing only in Appendix A are:

a_o - acceleration of the curve at the bow or stern.

a_1 - acceleration of the curve at the midship section.

f - flare of the unit ship.

F - flare of the actual ship.

m_o - section coefficient for zero flare.

m_s - section coefficient.

R - deadrise coefficient.

S - slope of the acceleration curve.

t - bow or stern tangent.

y_b - fraction of the midship beam.

Other symbols, not in general use, but appearing in this report, are defined when introduced.

I SUMMARY

A. Object:

The object of this investigation was to develop a more accurate and convenient method than is currently available for predicting the Curve of Statical Stability from preliminary design information.

B. Method:

Two unit parent hull forms were designed, using Taylor's Mathematical Lines as explained in Appendix A. The longitudinal coefficient, waterline coefficient, flare, and sheer were identical in both parent forms; but the block coefficient was varied. From each of the two parents, body plans were drawn, having systematically varied values of B/H and D/H. Then by the use of the integrator and "trial waterline" method, righting arms for all hulls were determined for inclinations of every fifteen degrees up to sixty. These data were then presented in the form of a family of curves.

C. Results:

The principal quantitative results are the family of stability and derived curves comprising Section IV of this report.

D. Conclusions and Recommendations:

1. Taylor's Mathematical Lines for hull design are a logical and useful approach to the investigation of stability, since by their use any independent coefficient or charac-

teristic of a hull can be varied by a known amount and the resultant change in statical stability known to be due only to that variation. It is, therefore, possible to investigate the influence upon the curve of statical stability of any of the hull coefficients and characteristics.

2. The ultimate objective of the investigation could not be fully attained in the time available by this thesis group, but the method and direction of future investigations have definitely been indicated.

3. It is recommended that the investigation be continued in its present form so that it will ultimately include hull forms based on at least four different values of l, four different values of b for each value of l, five values of B/H for each value of b, and four values of D/H for each value of B/H. To complete the study, it is recommended that the effects of flare, waterline coefficient, and sheer then be investigated in a similar manner.

4. It is further recommended that the complete data be presented in the form of families of curves so that the preliminary designer can enter with the basic hull coefficients and characteristics and, by means of a few interpolations, arrive at the curve of statical stability for his design.

II INTRODUCTION

This investigation, and the ones following it, are intended to be of use to the ship designer in the preliminary stage. As things stand now, he can find many things about a new hull without drawing a line or fairing a point. He can find its probable propulsion characteristics from the Standard Series, from a parent ship he can find its probable displacement, and from the duties of the design in service he can find its principal dimensions and coefficients of fineness. But even by laborious calculations for the height of the center of gravity he can get only an approximate value of the metacentric height, GM; and he can get no knowledge at all of the other stability characteristics. Except by inference from the stability curves of the parent, and from a general knowledge of the effect of GM on stability, the preliminary designer has no means of knowing:

1. The range of stability.
2. The angle of maximum righting arm.
3. The value of maximum righting arm.
4. Whether his curve of statical stability rises above the tangent at the origin, or falls below it.

Many attempts to correct this situation have been made. In 1920, Dr. Heinrich Schultz (1) gave analytical formulae for the ordinates of a curve of statical stability.

Aleman (2) applied these to merchant types and obtained good results. Guney and Unel (3) demonstrated that they are not applicable to destroyer-type hulls, and developed an equation for the angle of maximum righting arm. Their formula is:

$$\sin \phi_m = K_3 (GM/BM)^{\frac{1}{2}}$$

where K_3 is a parameter depending on GM, BM, "virtual freeboard", and ϕ_m of actual ships. The formula is applied to a number of destroyer hulls in reference (3). The maximum error found was about -12% in 50°, but was not consistent in either magnitude or sense. Moreover, for the purposes of the preliminary designer, it requires exact knowledge of both BM and GM, neither of which is readily available with any precision.

Hushing (4) used two destroyer hulls considered to be the usual extremes in this type of ship, and, using Benjamin's method of integration, obtained curves enabling the destroyer designer to approximate his curve of statical stability with fair accuracy, so long as the new hull does not depart radically from the usual destroyer lines.

All of these methods, for the purpose of the preliminary design, have certain defects:

1. They give too little information, or

2. They give results applicable to only one type.

In short, they attack only the fringes of the problem.

Latimer and Ramsey (5), and McKay (6), have gone to the center of the problem by using hulls of geometrical form

(a rectangle, a triangle, and an ellipse). They obtained good correlation with ships in service. The stability curve obtained by their method is always on the conservative side, and the error in the value of maximum righting arm is less than 9%. Moreover, their method has the advantage that a designer need know only his principal dimensions and coefficients of fineness to apply it.

The authors felt that this was the proper approach, but that the geometrical hull was an artificiality that introduced unavoidable error into the final results. It was suggested by Professor G. C. Manning of the Massachusetts Institute of Technology that this objection could be removed by using actual ship-shaped forms. For a time it was considered that correlation of stability curves with various parameters could be obtained by study of the stability of ships in service, but this idea was abandoned in short order. It is simple enough to find the variation in the curve of statical stability between two ships, but it is impossible to know what caused it unless only one parameter has varied. This is normally not the case, of course, but is perfectly possible with mathematical lines. It was, therefore, decided to use Taylor's Mathematical Lines.

The very word "mathematical" immediately suggests the possibility of using an exact integration by the calculus, rather than any of the numerous mechanical or approximate

methods available. Time was spent in exploring this idea, but the method was abandoned for the reasons given in Appendix "B".

Eventually it was decided to use the standard integrator method of determining stability. This method is admittedly not accurate, since it presupposes that the ship does not trim as it heels, which we know it does. Nevertheless, it has merit in that it is used almost exclusively for determining stability (unless a model is inclined), and thus results by this method have comparative value. The usual method is to obtain Cross Curves of Stability, and from them, the Curve of Statical Stability. To do this, however, would invalidate the results: as the waterline on a hull is moved--as it must be to get cross curves--it would be unusual indeed if the values of the coefficients of fineness did not change, and it is apparent that the values of D/H, B/H, and L/H must change. Conclusions arrived at on the basis that the hull had a given block coefficient, then, would be dubious at best. Moreover, if, after arriving at the cross curves, we select only one displacement (that at the Designer's Water Line), we have done a great deal of work which is of absolutely no use to us. In addition, the accuracy of the value we get from the cross curves is dependent on the manner of their fairing, and these curves are notoriously difficult to fair. For these reasons, it was decided to use only one displacement, and to obtain the Curve of Statical Stability directly. The method of "trial waterlines" used is explained under "Procedure".

Now, the parameters to be used in the presentation of the results are a matter of considerable interest. The variables entering into the problem are:

Length

Beam

Draft

Depth

Righting Arm

Area of Waterplane

Displacement

Angle of Inclination

All of these variables have the dimensions of $(\text{feet})^0$, $(\text{feet})^1$, $(\text{feet})^2$, or $(\text{feet})^3$, and any attempt at dimensional analysis will afford only the usual coefficients of fineness, and various ratios between the linear dimensions: D/H, B/H, GZ/B, etc. Of necessity, then, we must choose among them. It will be noted from the results presented that the variable "L" does not appear as long as the effect of trim is not considered. Neither was it necessary to consider the displacement. This is discussed at more length in Appendix "B".

A definite plan of variation was followed, which is treated more fully in "Procedure". Briefly, the ratio of D/H was handled merely by reducing the freeboard at constant draft, and the ratio of B/H by changing the vertical scale.

In attacking this investigation, certain simplifications were used:

1. KG = H.
2. Stations #0 and #10 have zero area.
3. No half-siding.
4. No camber.
5. No sheer.
6. Certain values of flare were used (see Appendix "A").
7. The above-water body of Hull Series "A" was faired in by eye, the offsets taken and recorded, and again used in Hull Series "B". They are listed under "Procedure".
8. The curve of statical stability is believed to be of little use for angles greater than the angle of maximum righting moment. The authors, therefore, chose to limit their investigation to angles of inclination less than 60°. By using intervals of 15°, this affords five points on the curve of statical stability, and allows better definition of that part of the curve of primary interest to the designer.

III PROCEDURE

After a basic hull form was designed by the use of Taylor's Mathematical Lines, as described in Appendix "A", a body plan was drawn using the offsets so obtained.

The body plan was plotted on cross-section paper with a beam of ten inches, about the largest value for which the body plan can be continuously integrated by the Amsler integrator. When the body plan is drawn to this scale, the offsets can be plotted to two significant figures, and the third figure may be approximated.

The above-water body was arbitrarily faired to the deck edge on the unit body plan (B/H=2) of Hull Series A for a value of the ratio of depth to draft of 2.0. In doing this an attempt was made to represent the above-water body of a ship of conventional type. The offsets as measured were later used for all subsequent ratios of B/H and on all body plans of Hull Series B.

Table of Offsets for Above-Water Body

WL / STA	12'	14'	16'	18'	20'
1	.420	.452	.490	.543	.620
2	.730	.763	.795	.833	.870
3	.900	.908	.920	.935	.960
4	.990	.992	.995	.997	1.00
5	1.00	1.00	1.00	1.00	1.00
6	.995	.997	.999	1.00	1.00
7	.975	.984	.988	.992	.992
8	.950	.967	.972	.967	.960
9	.830	.880	.902	.905	.890

In varying the ratio of beam to draft, (B/H), the actual value of beam was kept constant at ten inches and the draft changed. The values of B/H for which individual hull forms were drawn and integrated are: 2, 2½, 3, 3½, & 4.

For each ratio of B/H, integrations were performed for the following ratios of depth to draft, D/H: 1.4, 1.6, 1.8, & 2.0. Draft was maintained constant and the ratio varied by re-drawing the main deck for the value of D/H desired.

Since an inclined waterline drawn through G would in general not give the desired constant value of displacement, a trial waterline was drawn parallel to the inclined waterline through G and the volume of displacement compared with that for the uninclined ship. Successive trial waterlines were similarly drawn until the desired displacement had been obtained on two successive trials. The readings of the moment wheel were recorded for all integrations for which the trial area readings were usefully close to the desired value. Displacement was required to be within one percent.

The integrations were performed for use with Simpson's First Rule for both moments and volumes. All odd-numbered stations and all even-numbered stations were integrated continuously and the results tabulated for convenience on forms (see Figure a).

HULL SERIES ________ H/H ________

$\theta =$ ________ $D/H =$ ________

ODD STATIONS		EVEN STATIONS	
A	M	A	M

ODD STATIONS		EVEN STATIONS	
A	M	A	M

ODD STATIONS		EVEN STATIONS	
A	M	A	M

ODD STATIONS		EVEN STATIONS	
A	M	A	M

$$\frac{GZ}{B} = \frac{\Sigma f(M)}{5 \times \Sigma f(A)} = \underline{\qquad\qquad}$$

Fig. (a)

It was seldom necessary to perform a complete integration to establish a close approximation to the final position of a given waterline. If, when integrating for a trial waterline, the area of the even-numbered stations compared closely to the area of the even stations for the uninclined ship, the total area would be equally close to the desired value. This procedure decreased the work of determining the position of the inclined waterline. One can expect to arrive at the proper displacement on the third attempt.

Since the integrations were performed about a vertical plane through G, the restoring moment is given by:

$$M = \frac{1}{3} s \sum f(M)$$

and the volume of displacement by:

$$V = \frac{1}{3} s \sum f(A)$$

When using the Amsler integrator in conjunction with Simpson's Rule and remembering that first and last stations have zero area,

$$\sum f(M) = 40 \, (\text{scale factor})^3 \left[2\sum M_o + \sum M_e \right]$$

$$\sum f(A) = 20 \, (\text{scale factor})^2 \left[2\sum A_o + \sum A_e \right]$$

$$\therefore \; GZ = \frac{2 \, (\text{scale factor}) \left[2\sum M_o + \sum M_e \right]}{2\sum A_o + \sum A_e}$$

$$B = 10 \, (\text{scale factor})$$

$$\therefore \frac{GZ}{B} = \frac{2\Sigma M_o + \Sigma M_e}{5\left[2\Sigma A_o + \Sigma A_e\right]}$$

where

M_o = difference in readings of moment wheel for odd stations.

M_e = difference in readings of moment wheel for even stations.

A_o = difference in readings of area wheel for odd stations.

A_e = difference in readings of area wheel for even stations.

In order to determine the slope of the curves of statical stability in the vicinity of the origin, the value of GM/B was computed as follows:

$$GM = KB + BM - KG$$

$$\therefore \frac{GM}{B} = \left(\frac{KB}{H} \times \frac{H}{B}\right) + \frac{BM}{B} - \left(\frac{KG}{H} \times \frac{H}{B}\right)$$

KB was determined by a method similar to that used for determining GZ, and then checked by a numerical integration of offsets. The results of these two methods checked within one percent.

BM/B was determined from the following relation:

$$BM = \frac{I}{V} = \frac{2}{3}\int_0^L Y^3 dx$$

$$BM = \frac{\frac{2}{3} \times \frac{L}{10} \times \frac{1}{3} \times \Sigma f(Y)^3}{bLBH}$$

$$= \frac{\frac{2}{3} \times \frac{L}{10} \times \left(\frac{B}{2}\right)^3 \times \frac{2}{3}\left[2\Sigma y_{odd}^3 + \Sigma y_{even}^3\right]}{bLBH}$$

$$= \frac{1}{180} \times \frac{1}{b} \times \frac{B^2}{H} \times \left[2\Sigma y_{odd}^3 + \Sigma y_{even}^3\right]$$

$$\frac{BM}{B} = \frac{1}{180} \times \frac{1}{6} \times \frac{B}{H} \times \left[2 \sum y_{odd}^{3} + \sum y_{even}^{3} \right]$$

where Y = actual waterline offset.

y = waterline offset of the unit hull.

Using the computed values of GZ/B, the slope at the origin determined by GM/B, and the qualitative information derived from a determination of the approximate angle of deck edge immersion, curves of statical stability were plotted for each value of B/H and D/H. Derived cross-curves were obtained by plotting the results for each value of θ.

For the benefit of future investigators, the following information may be of assistance in planning the work: to plot and fair a body plan of the type used requires about two hours. After experience has been gained in estimating the position of the waterlines and in performing the integrations, the total time, including the subsequent calculations, for one value of B/H will be about eight to ten hours. This time is exclusive of the time which will be needed to compute the initial mathematical unit hull form (see Appendix "A").

The Figures b, c, d, & e which follow are included in this section to illustrate the type of body plan used in the investigation. They are reproductions of four of the ten body plans actually drawn and integrated. As mentioned above, body plans for different values of D/H were obtained merely by re-drawing the main deck at a different height.

Example Illustrating determination of $\frac{GM}{B}$

Hull Series A $\qquad \frac{B}{H} =$ [illegible] $\qquad \nu = 0.608$

Station	y^3	Station	y^3
1	.0607	2	.3357
3	.7097	4	.9527
5	1.000	6	.9732
7	.8903	8	.7661
9	.3890		

$\Sigma y_{odd}^3 = 3.0497 \qquad \Sigma y_{even}^3 = 3.028$

$$2\,\Sigma y_{odd}^3 + \Sigma y_{even}^3 = 9.127$$

$$\frac{BM}{B} = \frac{1}{180} \times \frac{1}{0.608} \times 4 \times 9.127 = 0.334$$

$$\frac{KB}{H} = 0.562$$

$$\frac{KG}{H} = 1.000$$

$$\frac{GM}{B} = \frac{0.562}{4} + .334 - \frac{1.000}{4} = \underline{\underline{0.224}}\ \#$$

Fig. 6

Fig. C

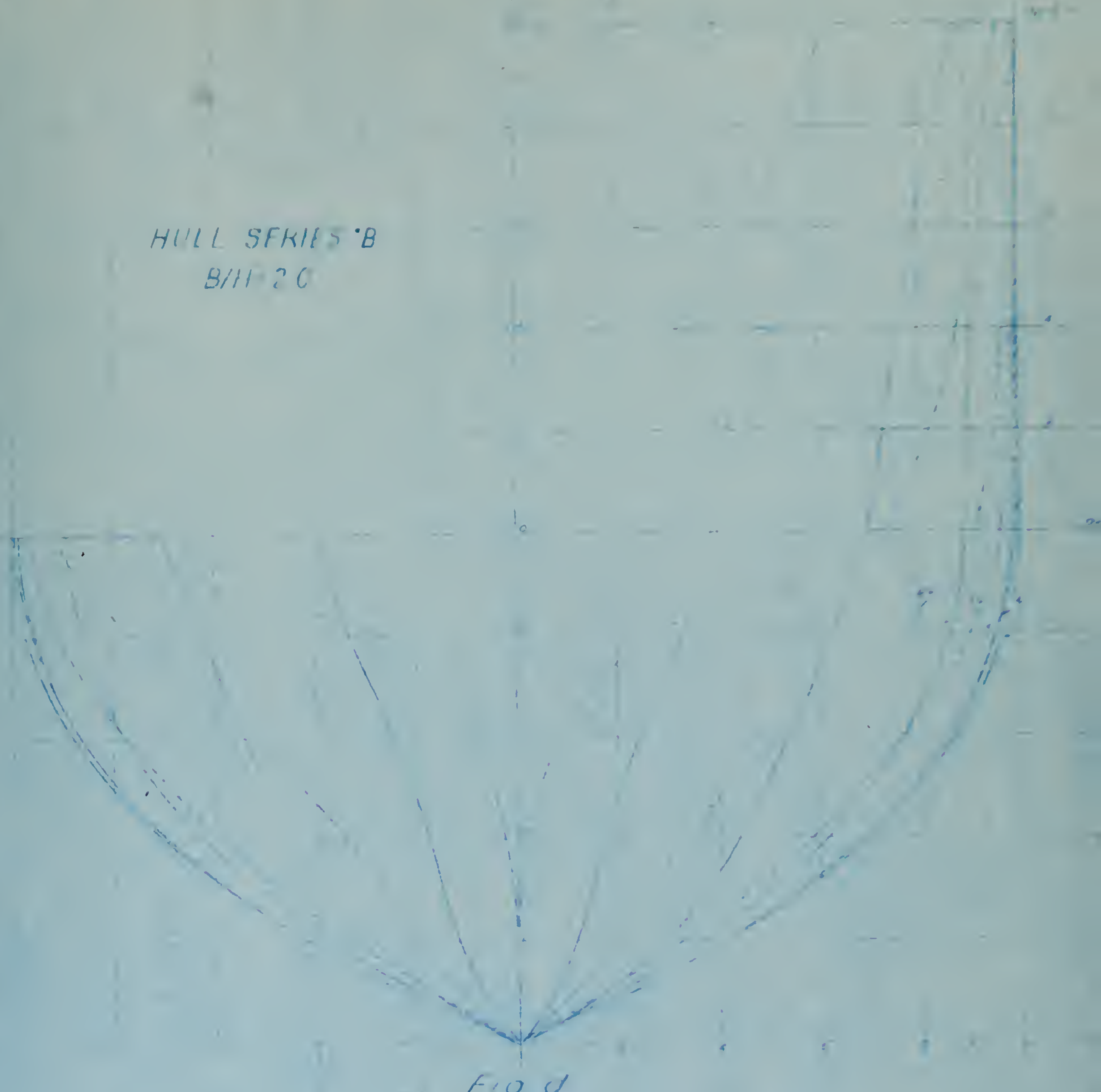


Fig. d

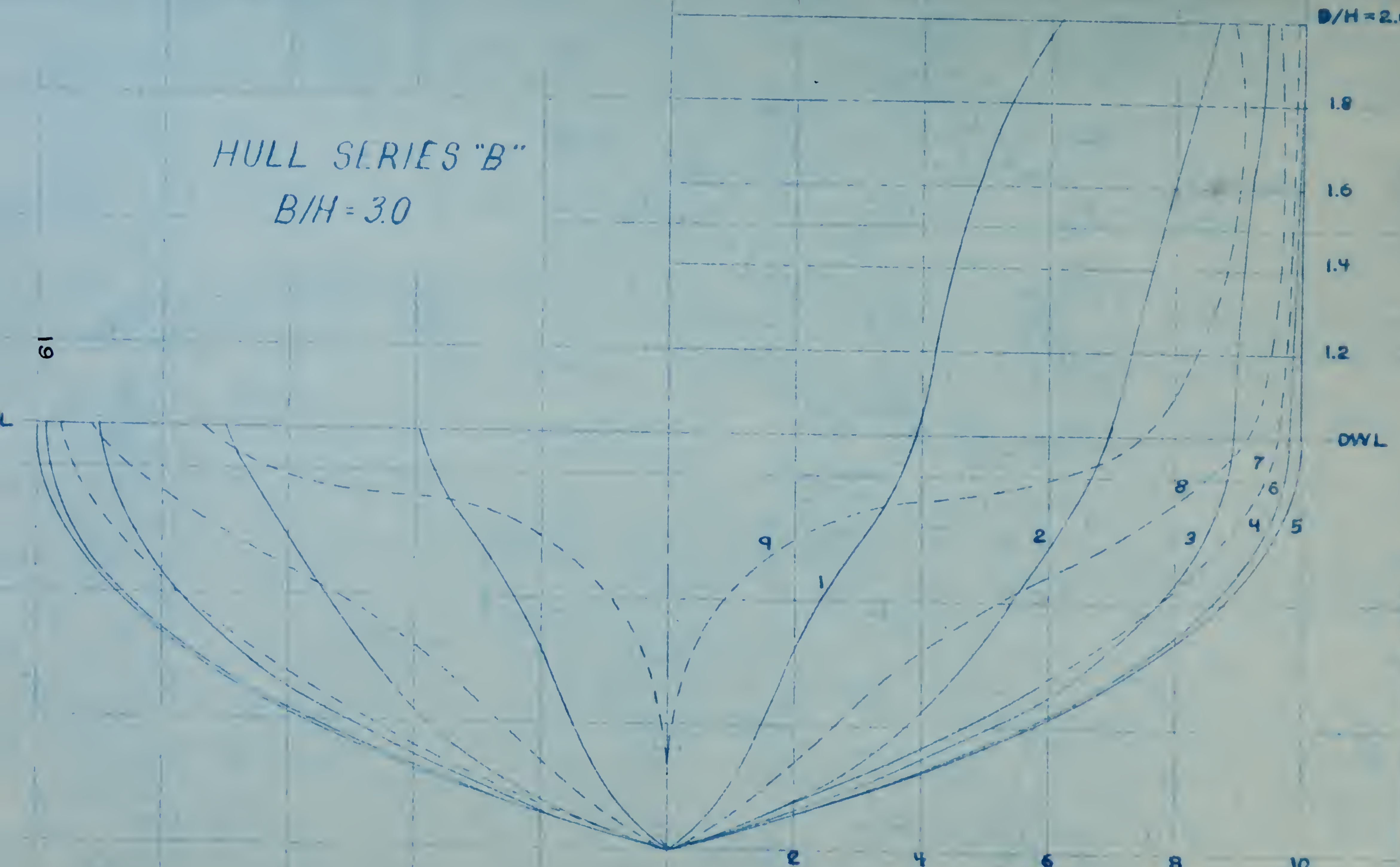


Fig. e

IV RESULTS

CURVES OF STATICAL STABILITY

FOR HULL SERIES A AND B

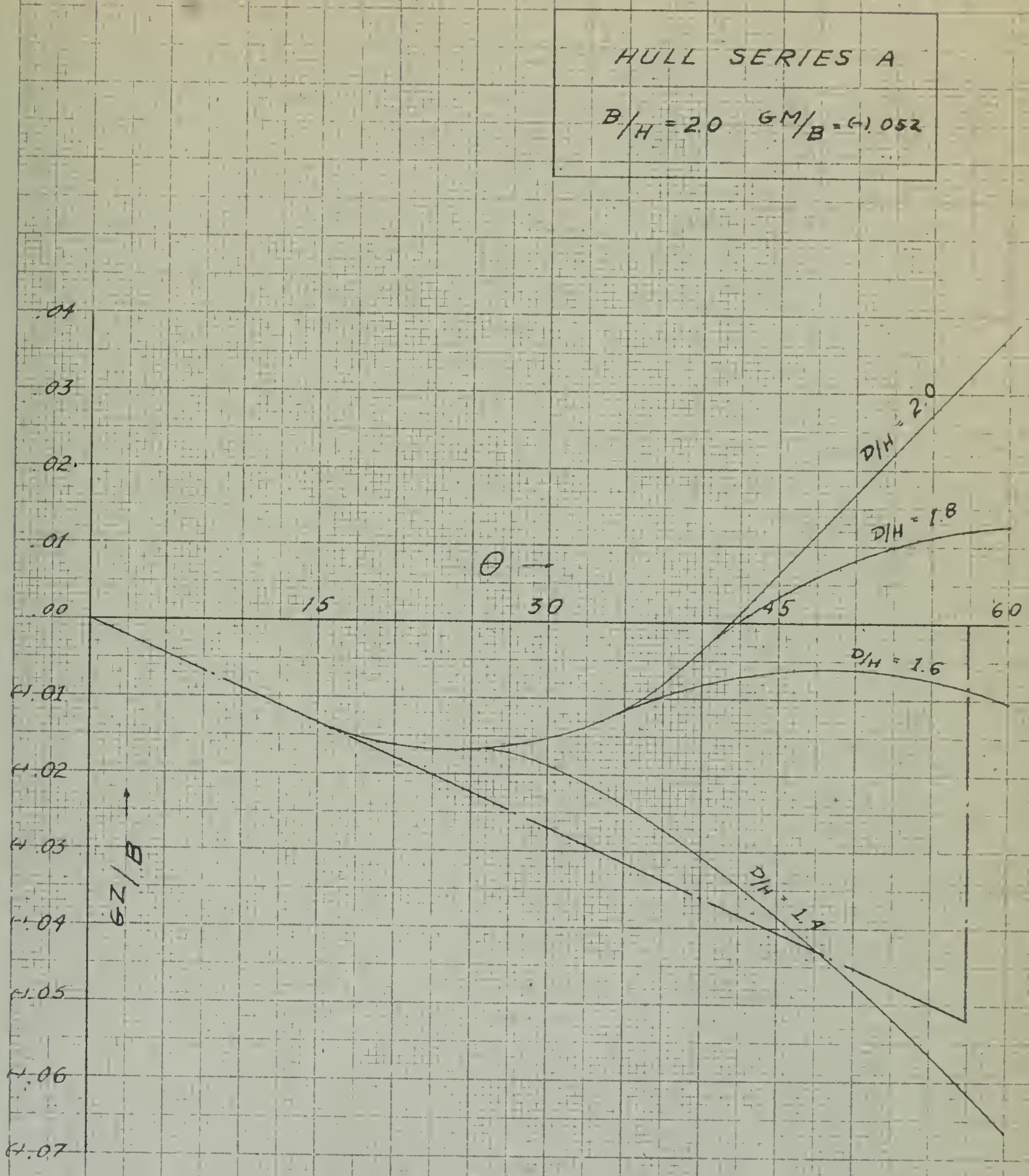

HULL SERIES A
B/H = 2.0 GM/B = -.052
.04
.03
.02
.01
0.0
-.01
-.02
-.03
-.04
-.05
-.06
-.07
GZ/B →
θ →
15
30
45
60
D/H = 2.0
D/H = 1.8
D/H = 1.6
D/H = 1.4

Fig. I

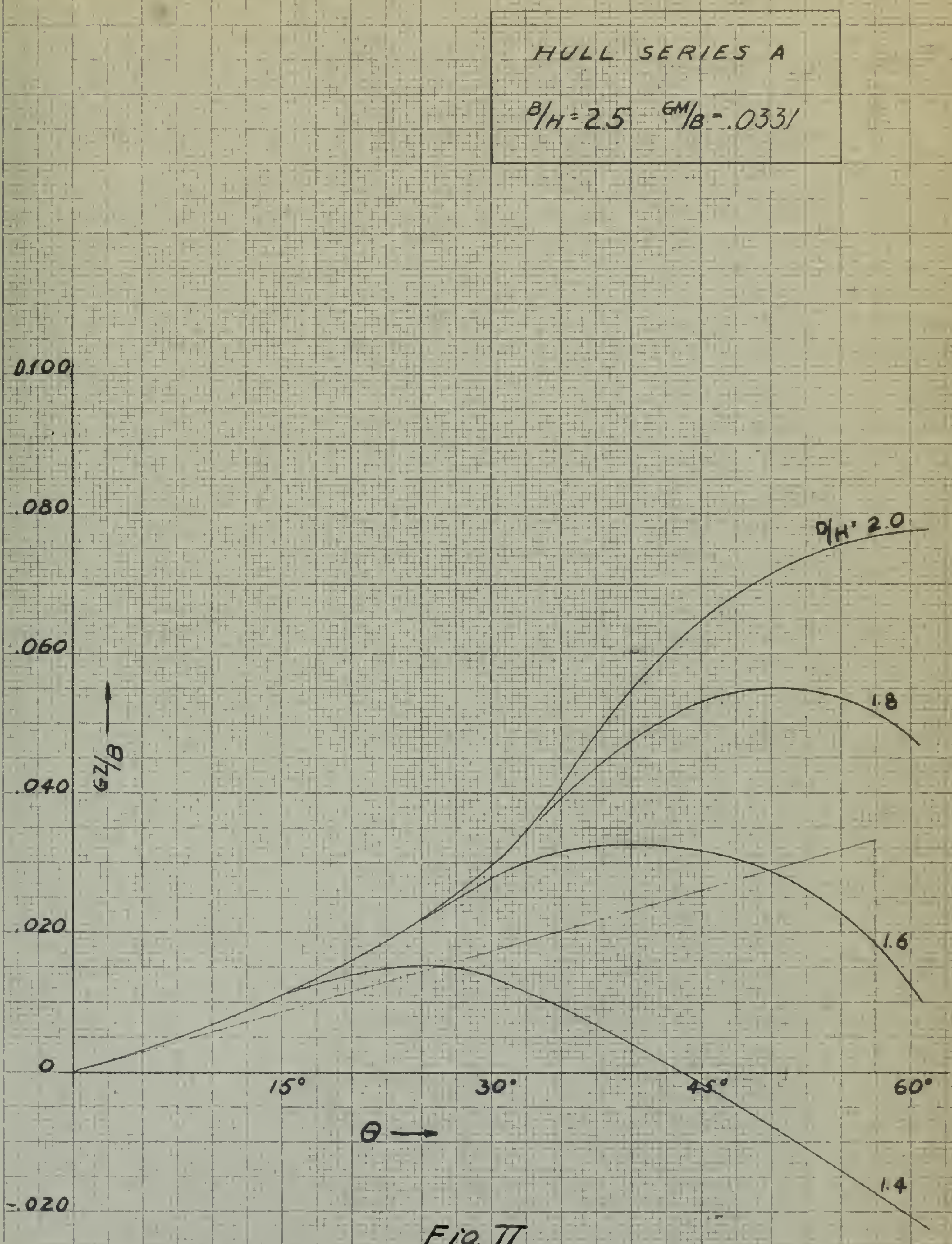

HULL SERIES A
B/H = 2.5 GM/B = .0331
0.100
.080
.060
.040
.020
0
-.020
GZ/B
15°
30°
45°
60°
θ
D/H = 2.0
1.8
1.6
1.4

Fig. II

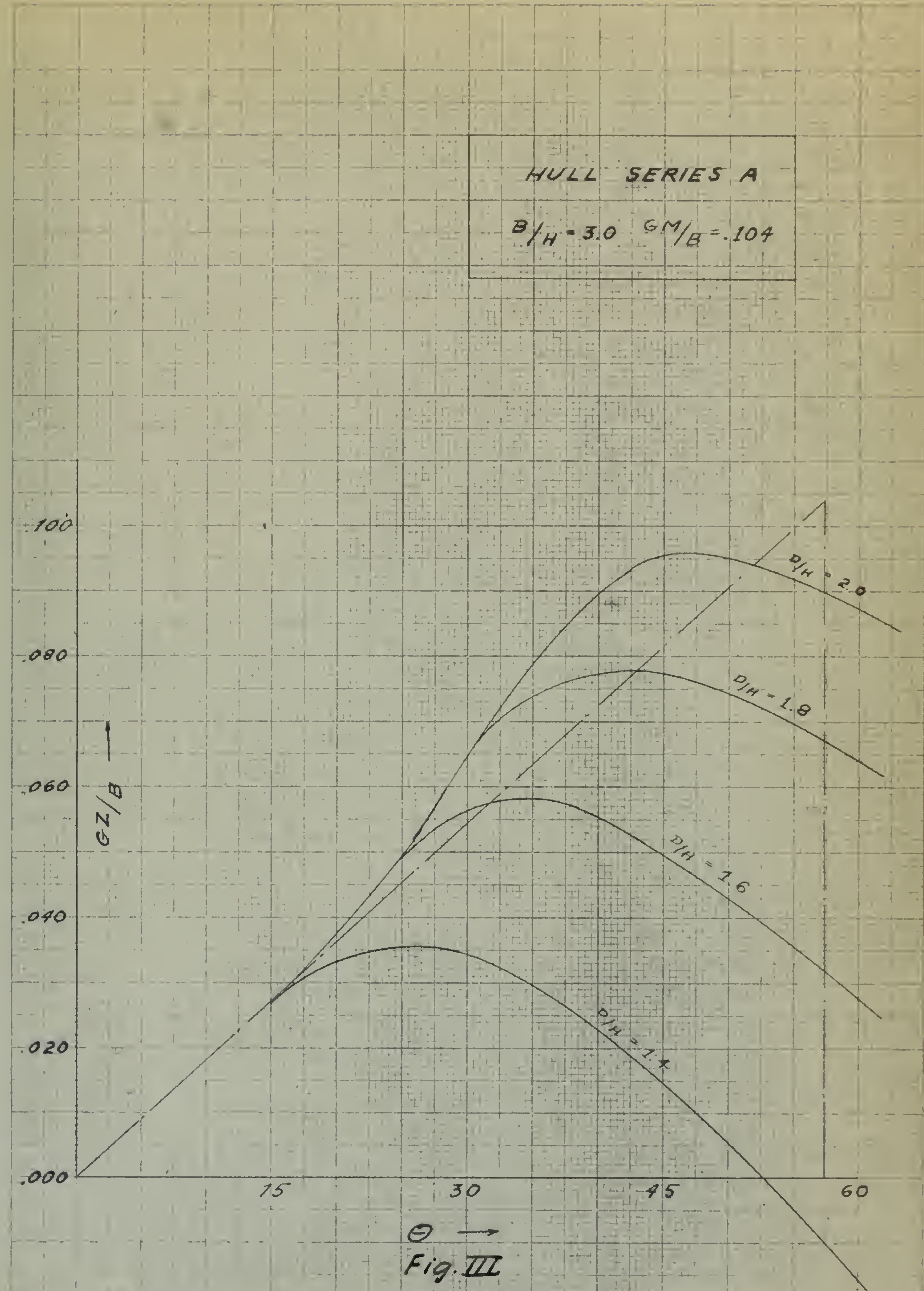
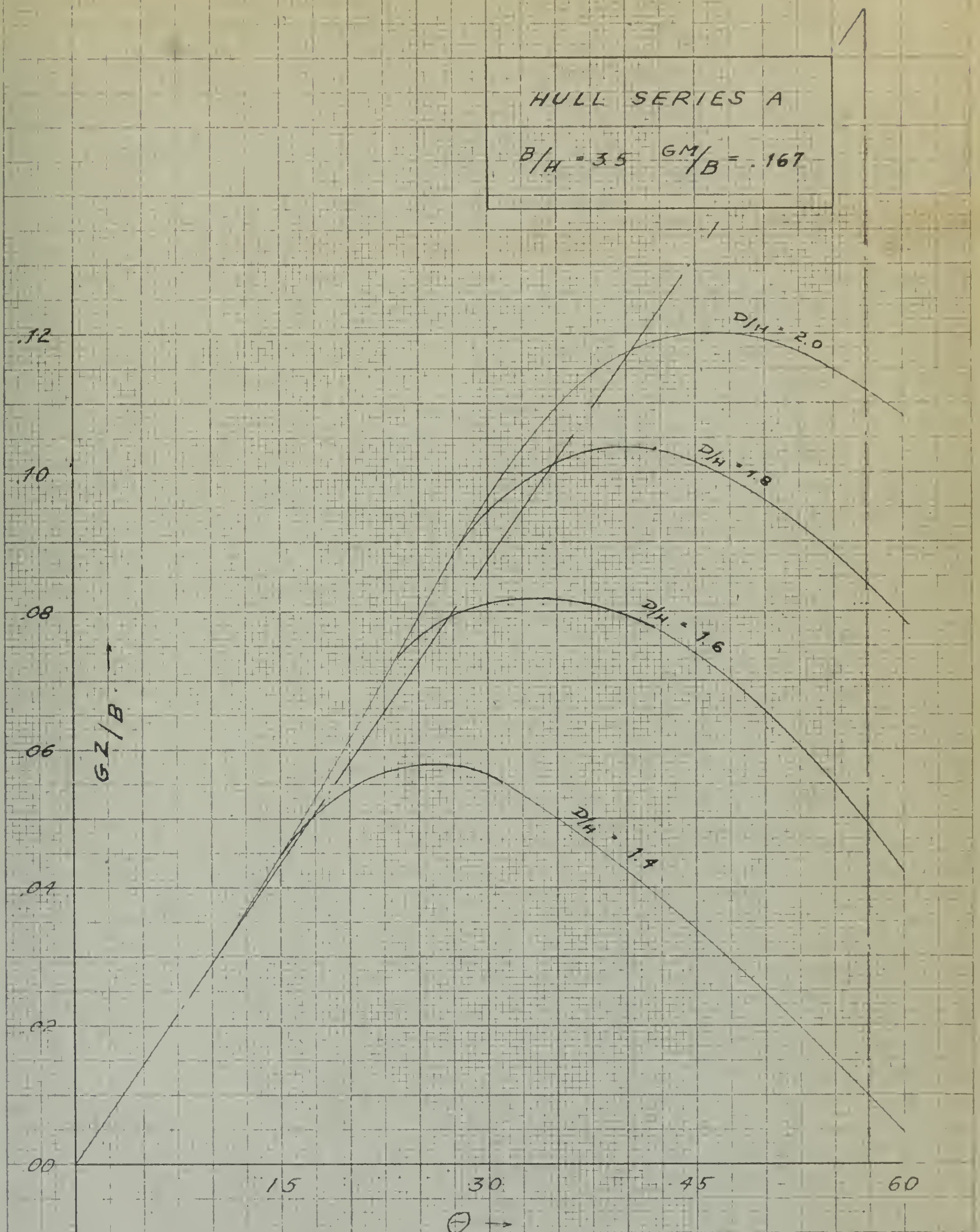

HULL SERIES A
B/H = 3.0 GM/B = .104
D/H = 2.0
D/H = 1.8
D/H = 1.6
D/H = 1.4
GZ/B
.100
.080
.060
.040
.020
.000
(-).020
15
30
45
60
Θ

Fig. III


HULL SERIES A
B/H = 3.5 GM/B = .167
D/H = 2.0
D/H = 1.8
D/H = 1.6
D/H = 1.4
GZ/B
.12
.10
.08
.06
.04
.02
.00
15
30
45
60
Θ
Fig. IV

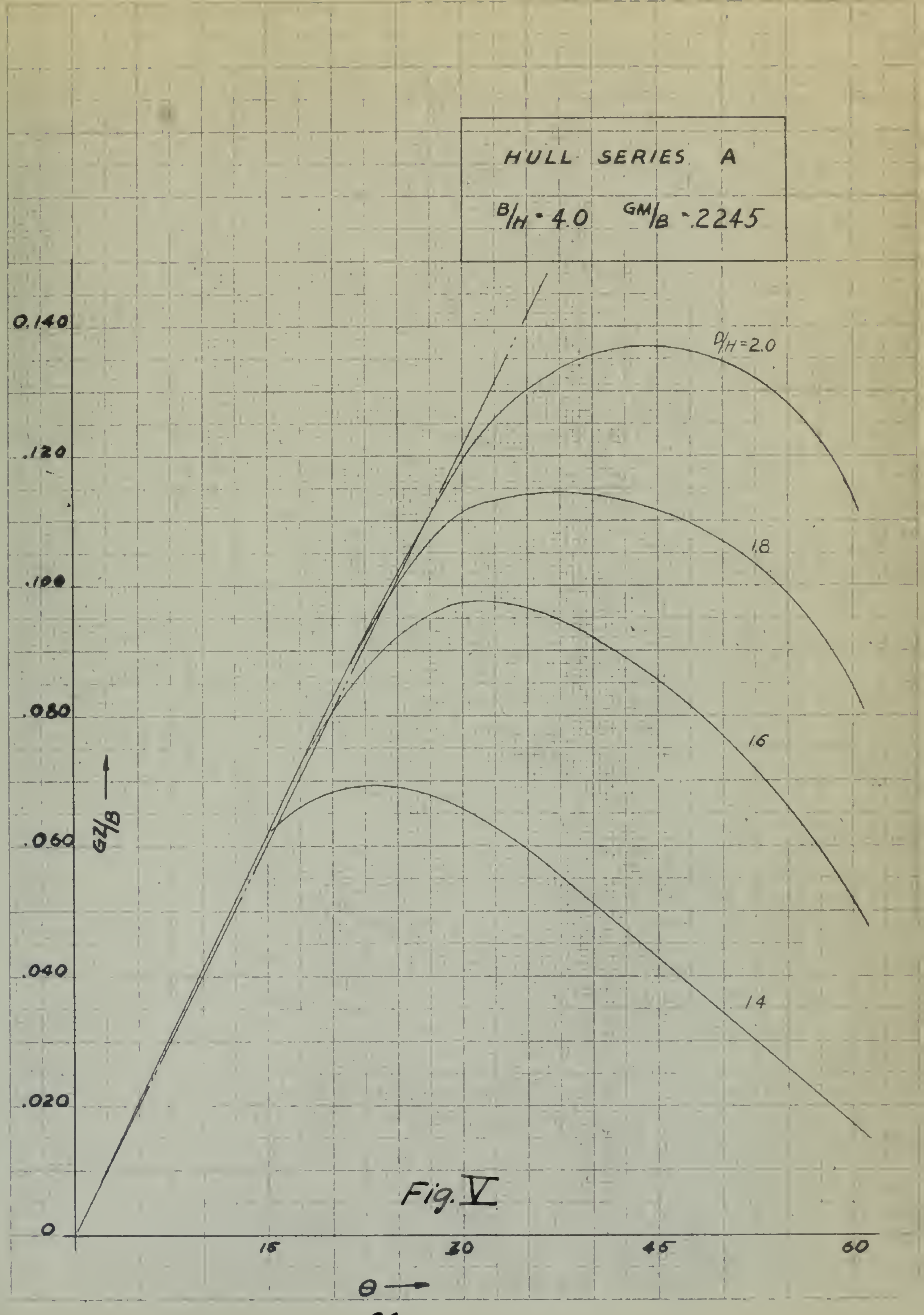

HULL SERIES A
B/H = 4.0 GM/B = .2245
0.140
.120
.100
.080
.060
.040
.020
0
GZ/B
D/H = 2.0
1.8
1.6
1.4
15
30
45
60
θ

Fig. V

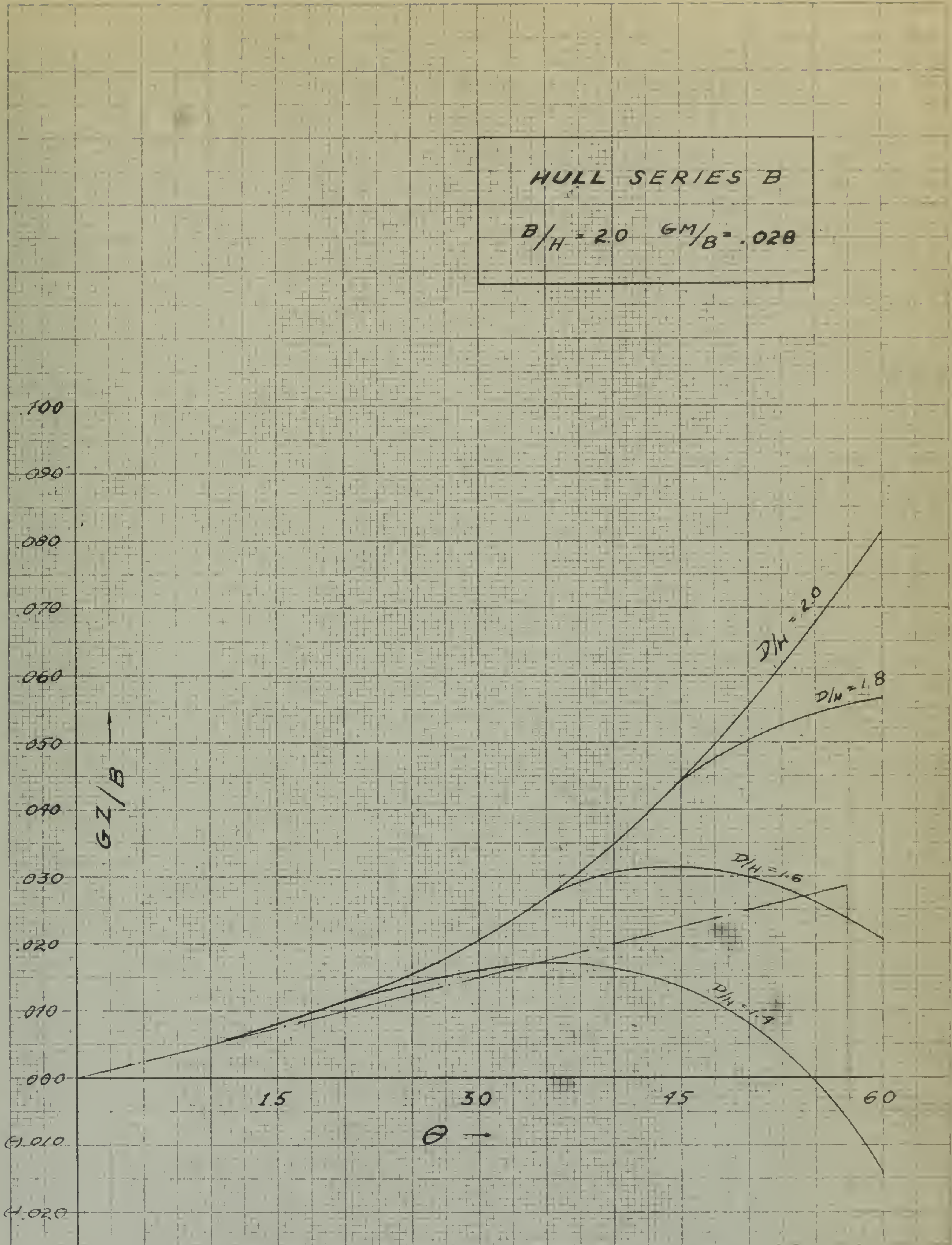

HULL SERIES B
B/H = 2.0 GM/B = .028
.100
.090
.080
.070
.060
.050
.040
.030
.020
.010
.000
(-).010
(-).020
(-).030
GZ/B →
15
30
45
60
θ →
D/H = 2.0
D/H = 1.8
D/H = 1.6
D/H = 1.4

Fig. VI

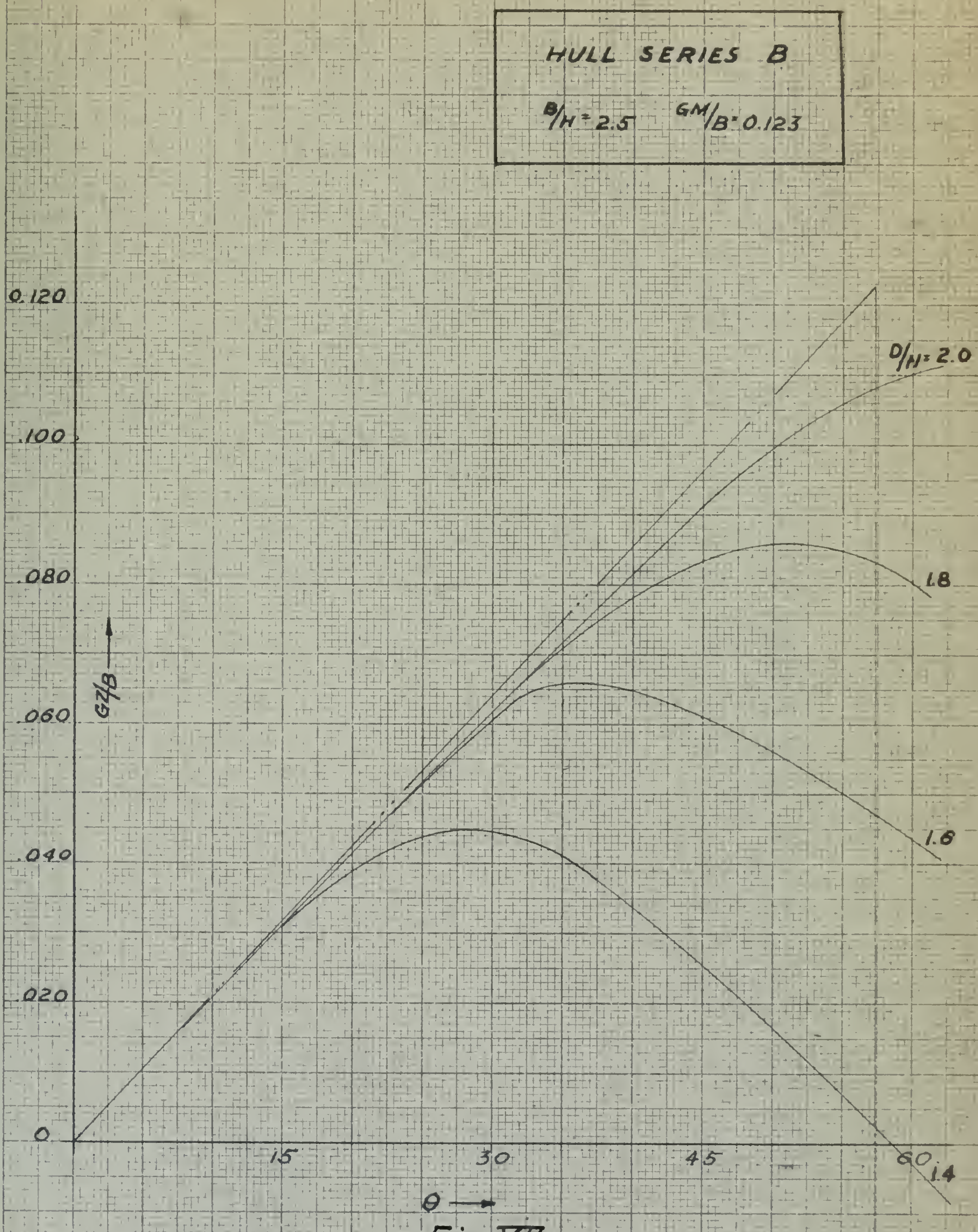
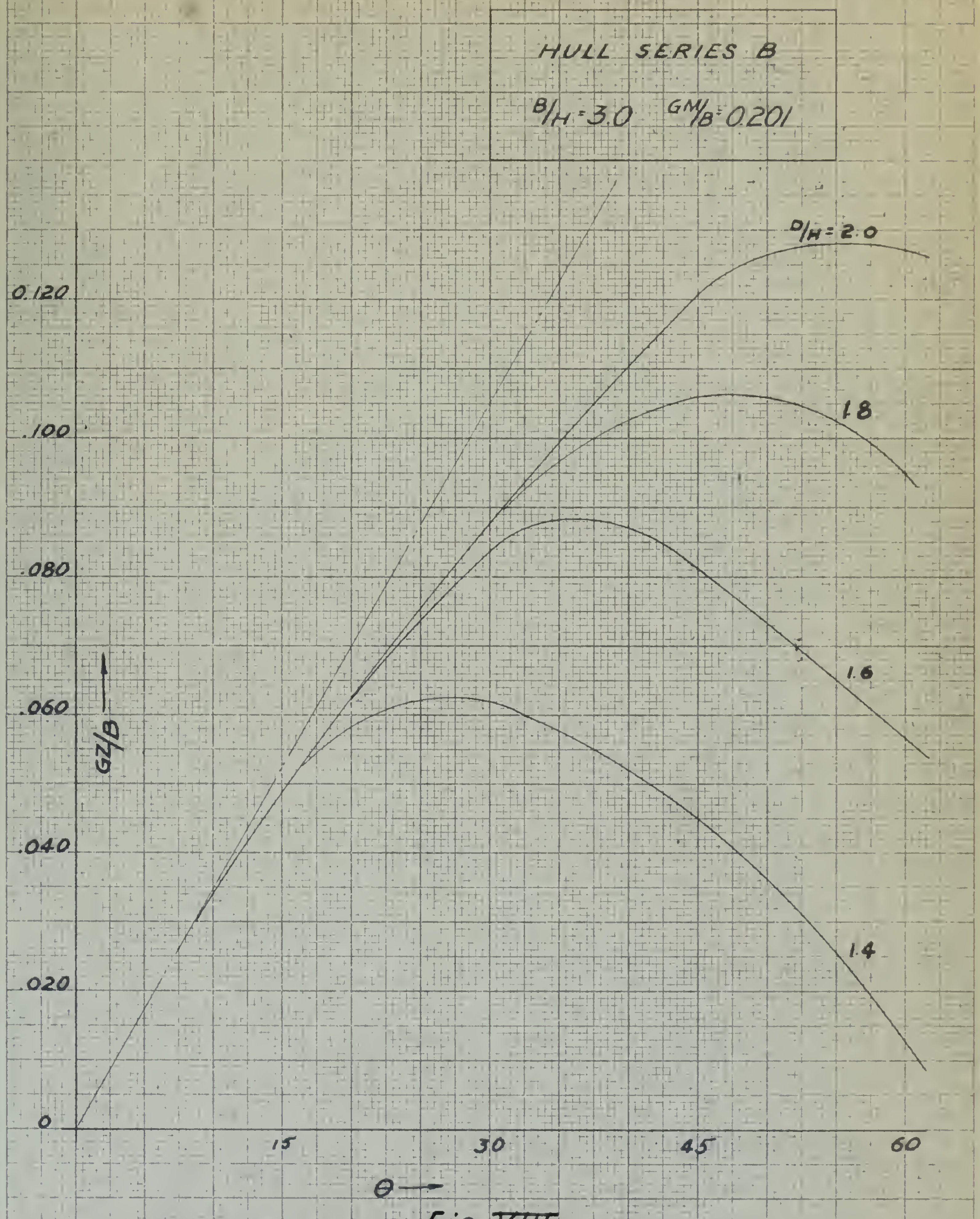

HULL SERIES B
B/H = 2.5 GM/B = 0.123
0.120
.100
.080
.060
.040
.020
0
GZ/B
15
30
45
60
θ
D/H = 2.0
1.8
1.6
1.4

Fig. VII


HULL SERIES B
B/H = 3.0 GM/B = 0.201
D/H = 2.0
1.8
1.6
1.4
0.120
.100
.080
.060
.040
.020
0
GZ/B
15
30
45
60
θ
Fig. VIII

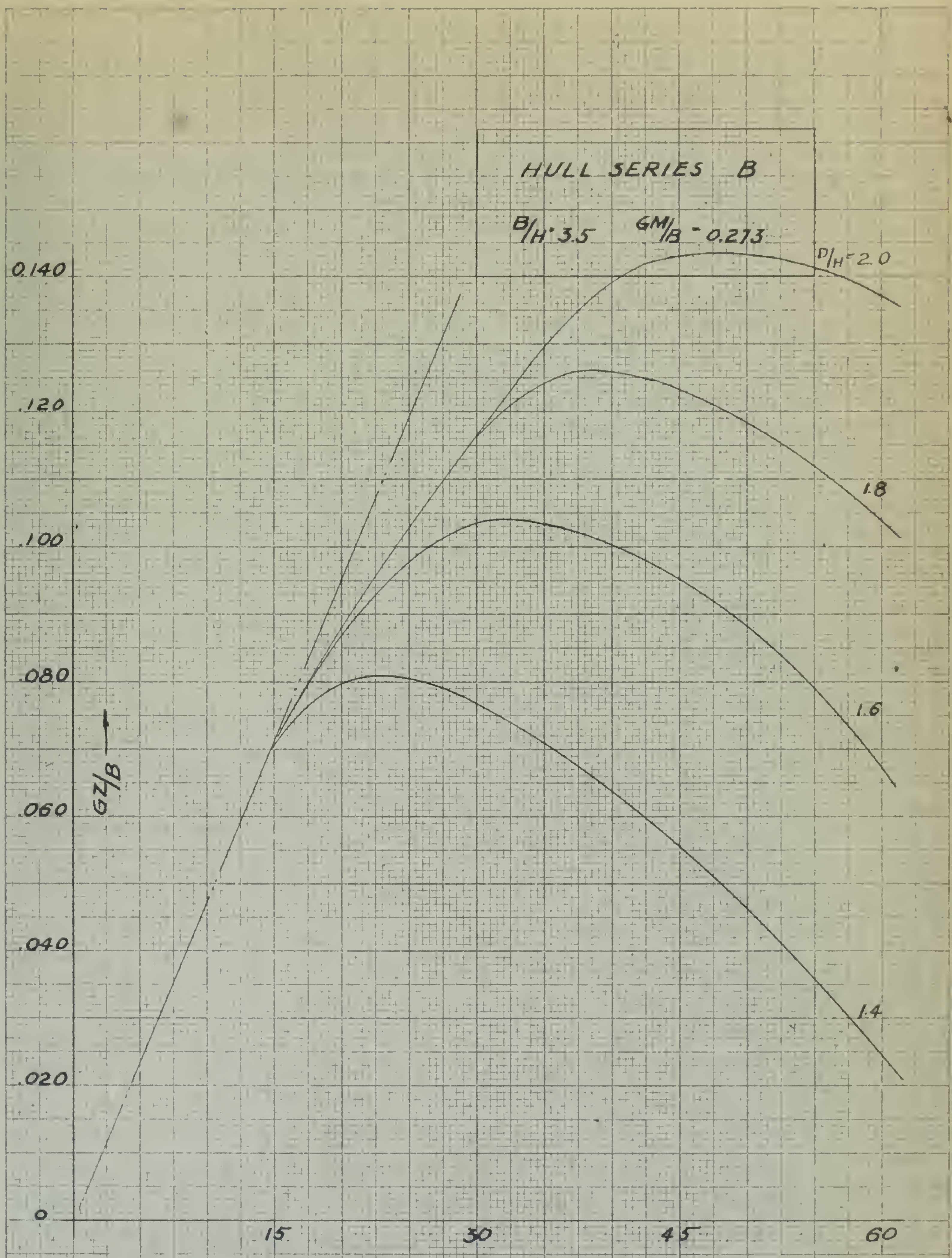

HULL SERIES B
B/H = 3.5
GM/B = 0.273
D/H = 2.0
1.8
1.6
1.4
0.140
.120
.100
.080
.060
.040
.020
0
15
30
45
60
GZ/B
θ

Fig. IX

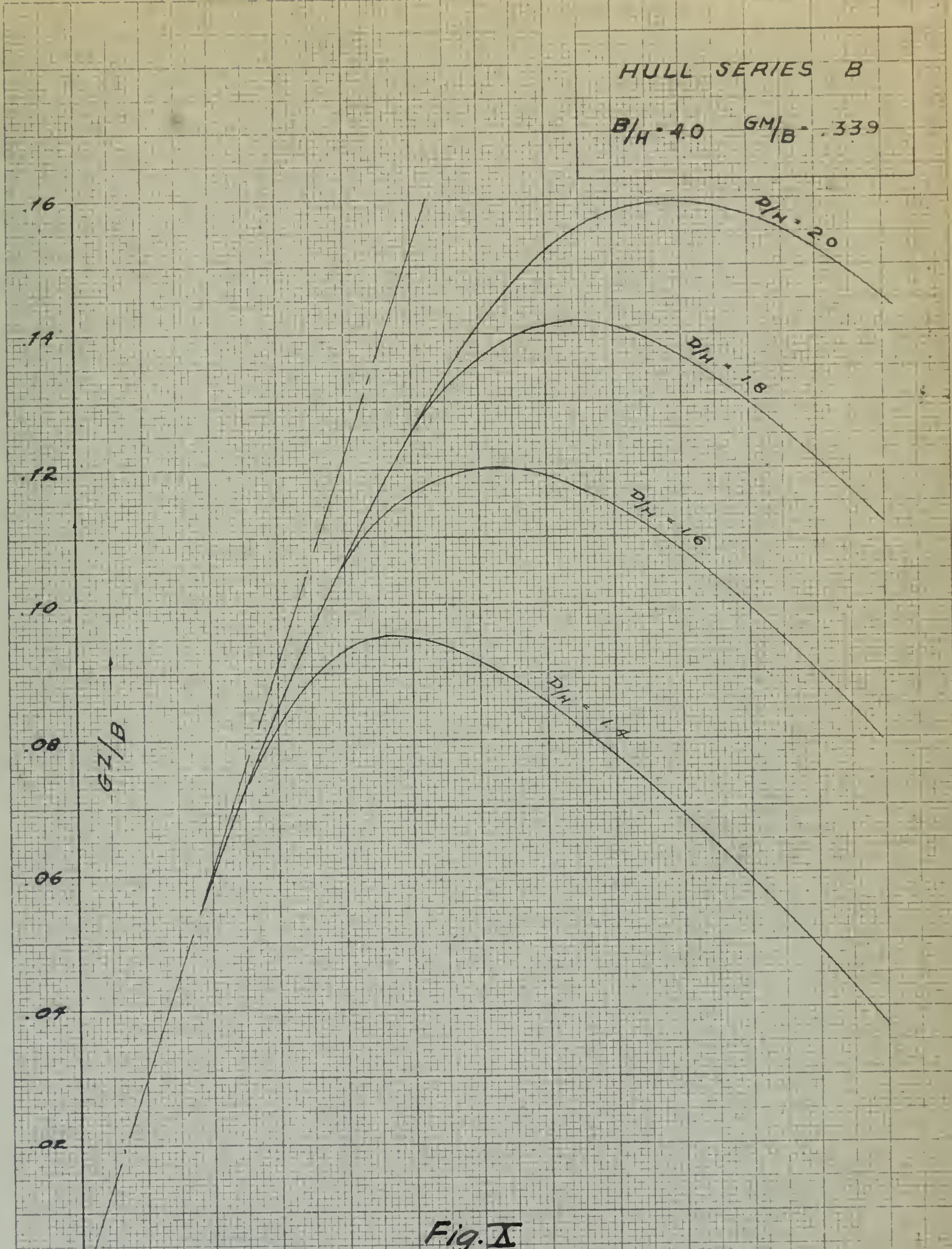
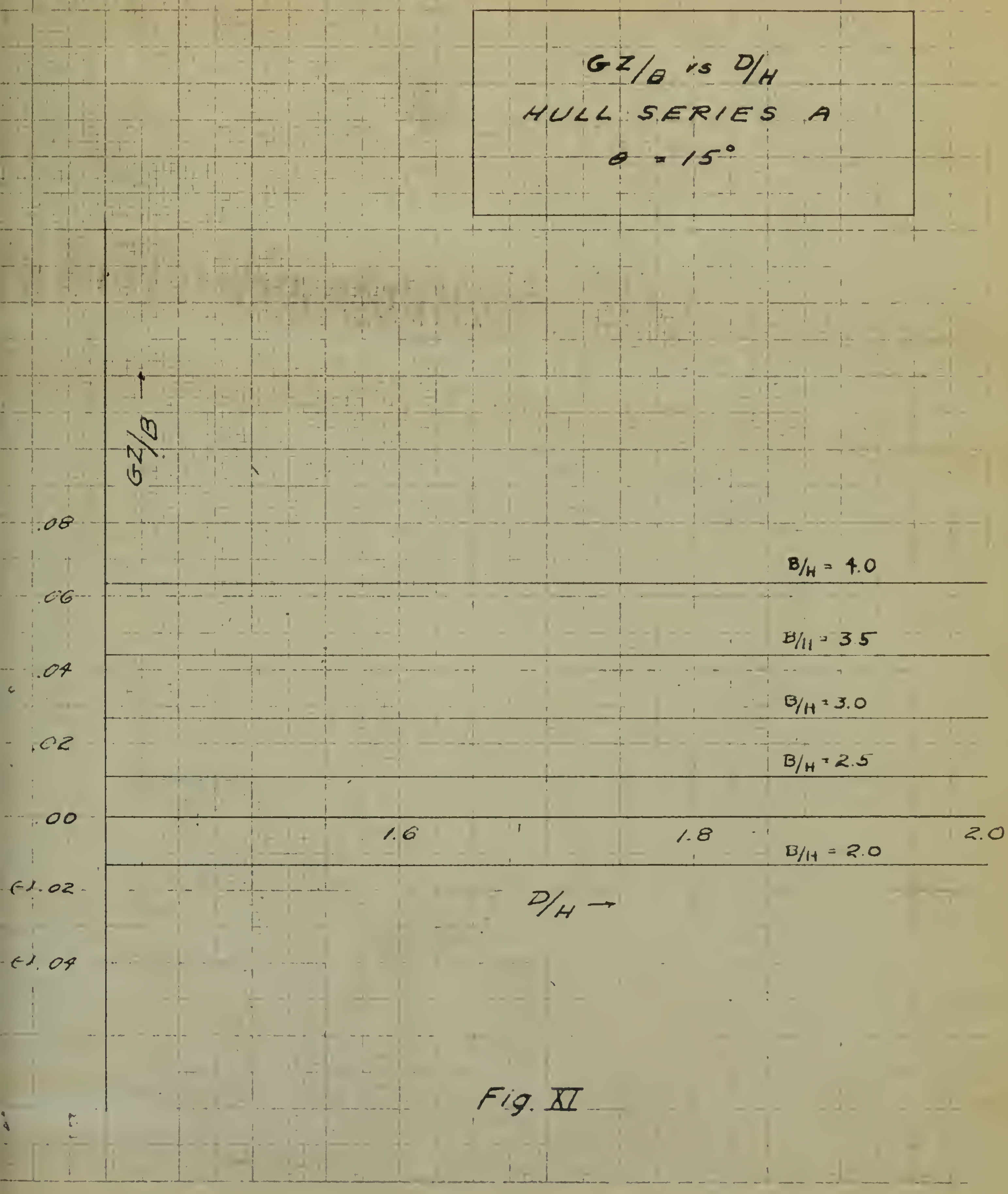

HULL SERIES B
B/H = 4.0 GM/B = .339
D/H = 2.0
D/H = 1.8
D/H = 1.6
D/H = 1.4
.16
.14
.12
.10
.08
.06
.04
.02
.00
GZ/B
15
30
45
60
θ

Fig. X

DERIVED CURVES OF RIGHTING ARM

PLOTTED FOR CONSTANT ANGLE OF INCLINATION

FOR HULL SERIES A AND B



GZ/B vs D/H
HULL SERIES A
θ = 15°
GZ/B →
.08
.06
.04
.02
.00
(-).02
(-).04
B/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
1.6
1.8
2.0
D/H →

Fig. XI

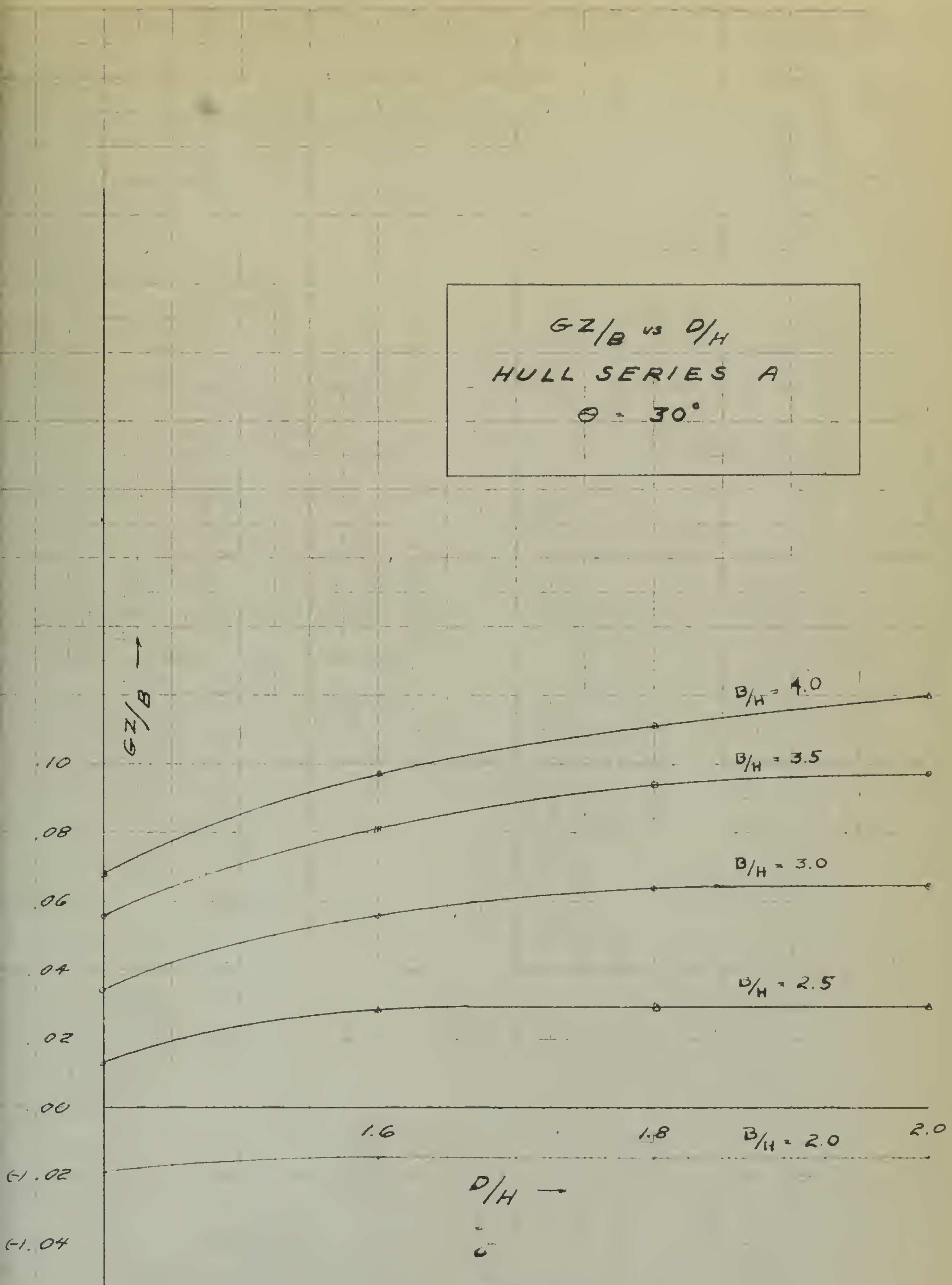
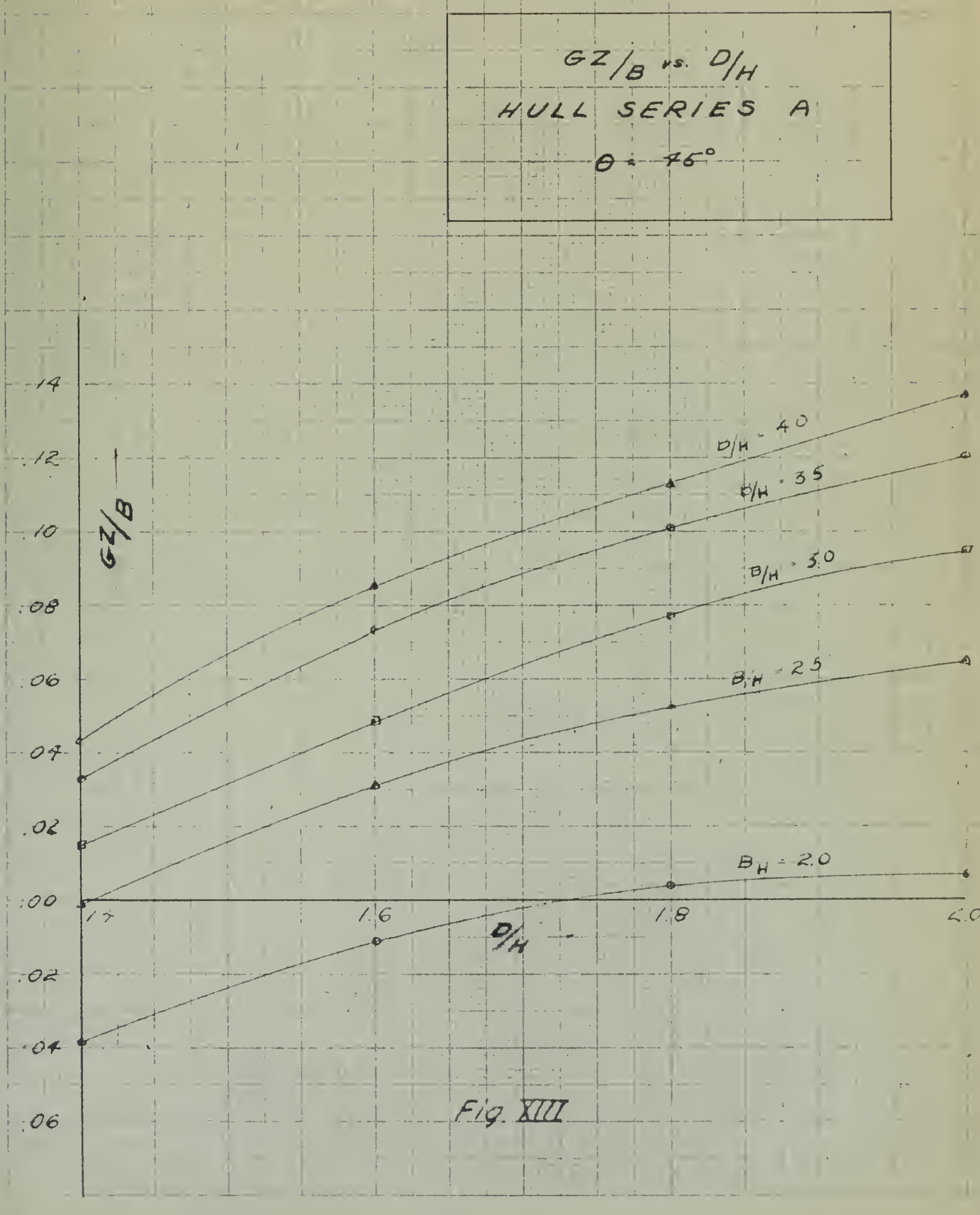
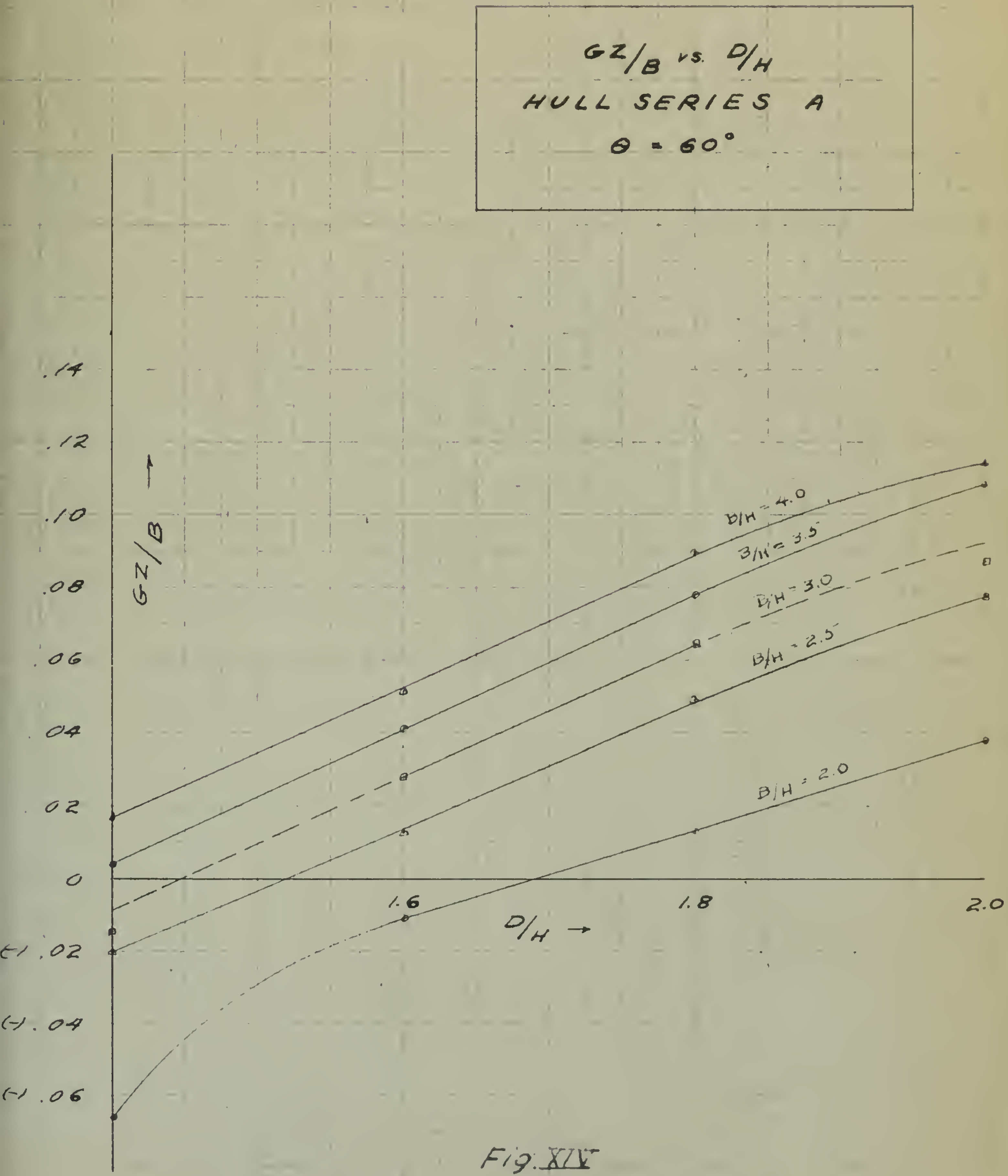

GZ/B vs D/H
HULL SERIES A
θ = 30°
GZ/B →
B/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
.10
.08
.06
.04
.02
.00
(-) .02
(-) .04
1.6
1.8
2.0
D/H →

Fig. XII



GZ/B vs. D/H
HULL SERIES A
θ = 45°
GZ/B
D/H
D/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
.14
.12
.10
.08
.06
.04
.02
.00
.02
.04
.06
1.4
1.6
1.8
2.0

Fig. XIII



GZ/B vs. D/H
HULL SERIES A
$\theta = 60°$
.14
.12
.10
.08
.06
.04
.02
0
(-) .02
(-) .04
(-) .06
GZ/B →
1.6
1.8
2.0
D/H →
$B/H = 4.0$
$B/H = 3.5$
$B/H = 3.0$
$B/H = 2.5$
$B/H = 2.0$

Fig. XIV

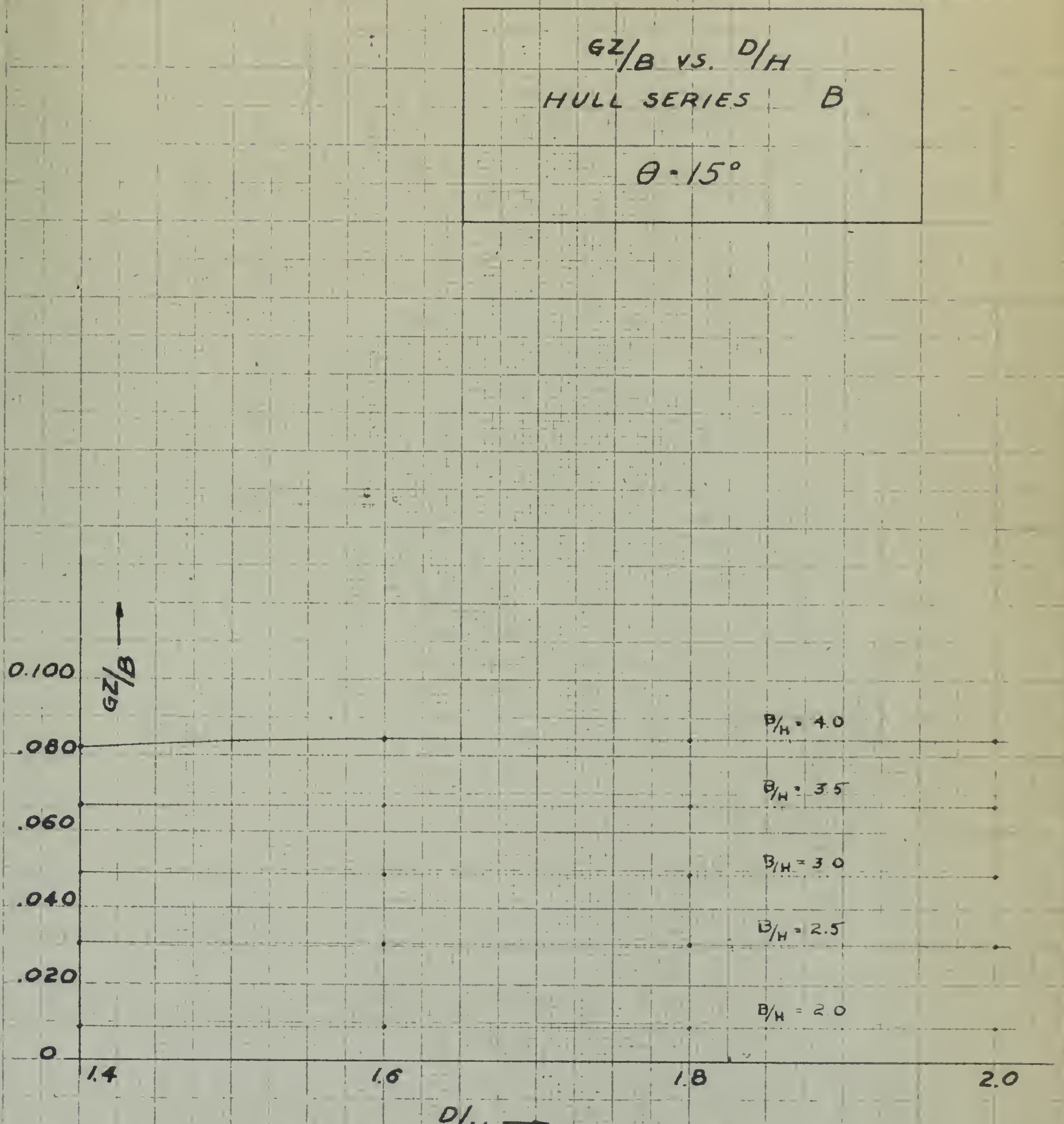

GZ/B vs. D/H
HULL SERIES B
θ = 15°
GZ/B
0.100
.080
.060
.040
.020
0
B/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
1.4
1.6
1.8
2.0
D/H

Fig. XV

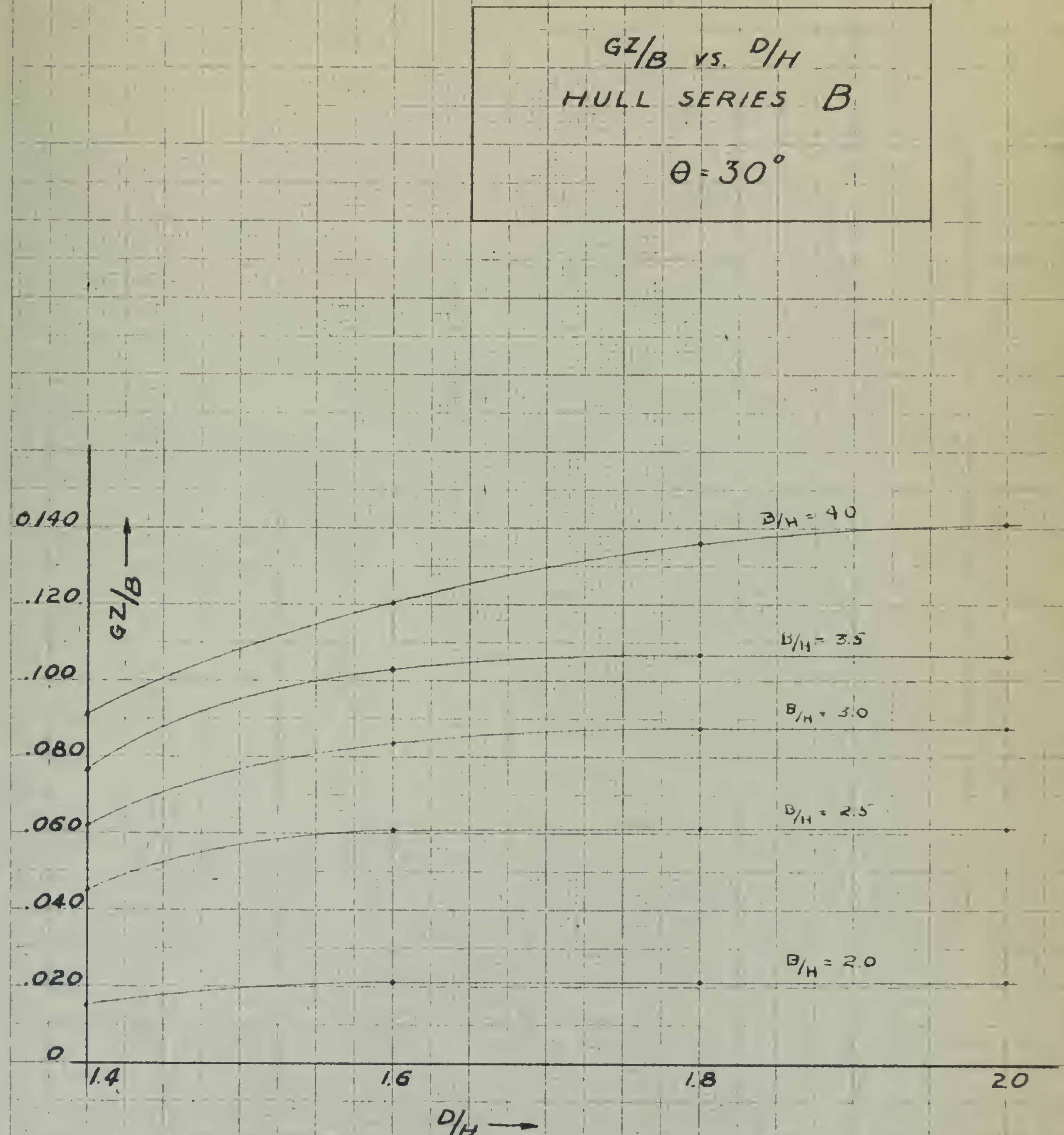
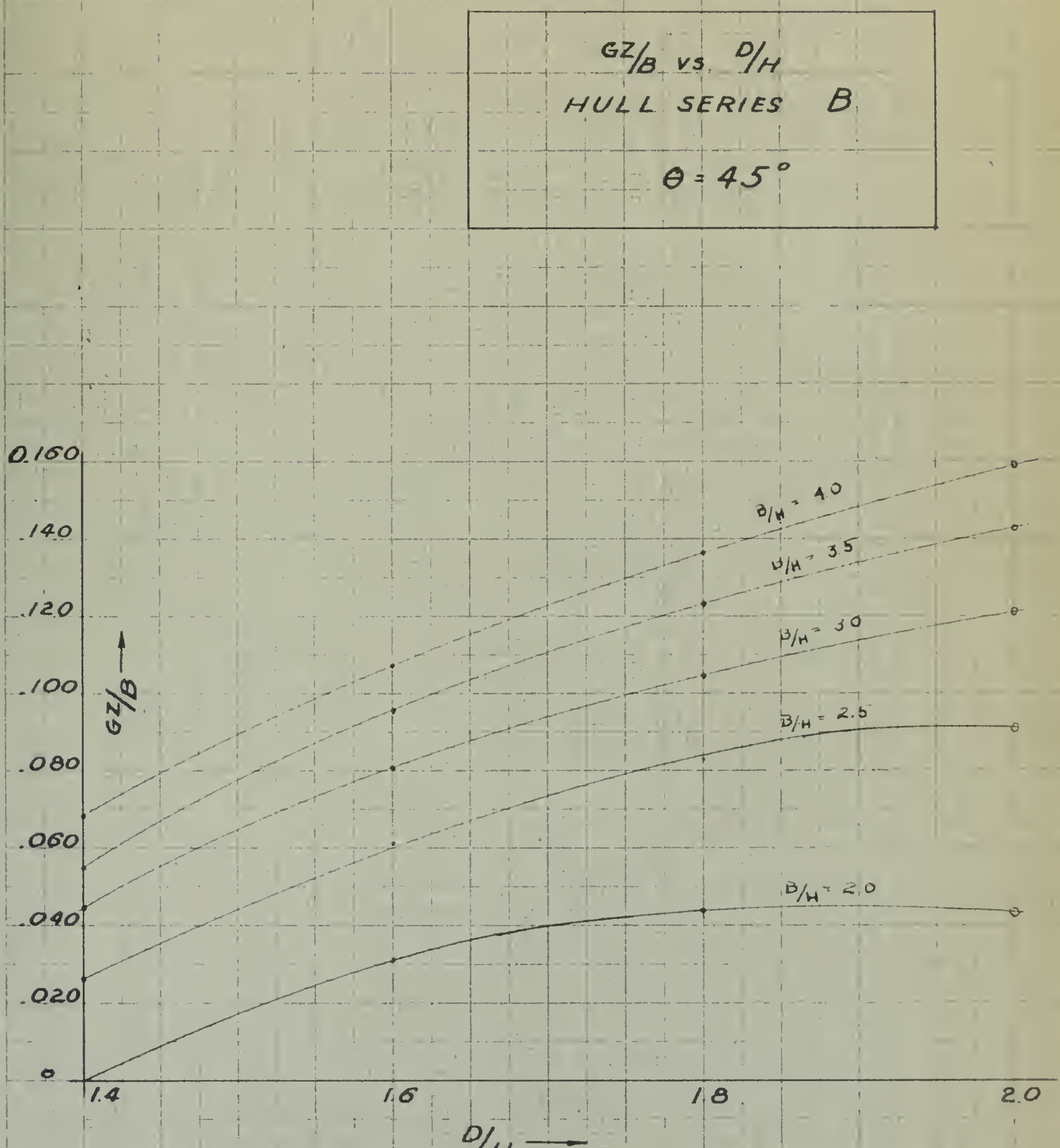
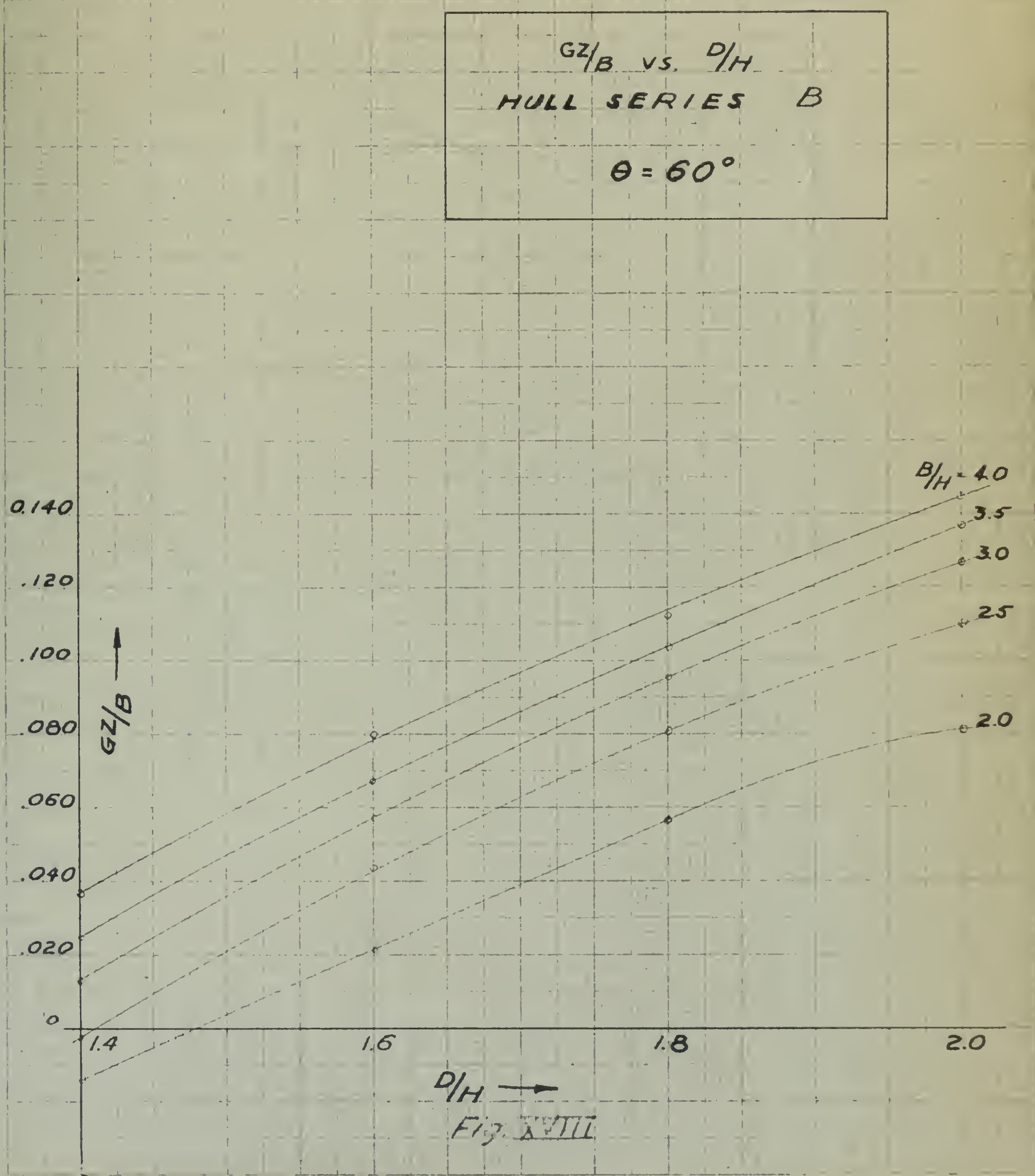

GZ/B vs. D/H
HULL SERIES B
θ = 30°
0.140
.120
.100
.080
.060
.040
.020
0
GZ/B
B/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
1.4
1.6
1.8
2.0
D/H

Fig. XVI



GZ/B vs. D/H
HULL SERIES B
θ = 45°
0.160
.140
.120
.100
.080
.060
.040
.020
0
GZ/B
B/H = 4.0
B/H = 3.5
B/H = 3.0
B/H = 2.5
B/H = 2.0
1.4
1.6
1.8
2.0
D/H

Fig. XVII


GZ/B vs. D/H
HULL SERIES B
θ = 60°
0.140
.120
.100
.080
.060
.040
.020
0
GZ/B
1.4
1.6
1.8
2.0
D/H
B/H = 4.0
3.5
3.0
2.5
2.0
Fig. XVIII

V DISCUSSION OF RESULTS

As set forth elsewhere, the object of the investigation was to develop a more accurate and convenient method than is currently available for predicting the curve of statical stability from preliminary design information. The authors believe that a long step has been taken toward this objective and that there has been indicated for future investigation a definite direction, which, if closely followed, will lead to the successful attainment of the objective.

The investigation thus far has demonstrated that the use of Taylor's Mathematical Lines for hull design is a logical method of obtaining the data necessary to show the quantitative effects, on the statical stability curve, of a known variation in any of the hull coefficients or characteristics. In this report, through the use of two hull series, there have been shown the quantitative effects of variations in block coefficient, beam-to-draft ratio, and depth-to-draft ratio. A continuation of the investigation along the same lines will be able to record the effects of changes in longitudinal coefficient and of further variations in block coefficient. It will also be perfectly feasible to obtain the effects of variations in sheer, flare, and waterline coefficient since a body plan evolved from Taylor's Lines is dependent upon these three characteristics as well as upon the other coefficients and characteristics previously mentioned.

After all the data derived from variations in l, b, B/H, and D/H have been collected and the quantitative effects compared, it will be a simple matter to present the information in a form readily usable by preliminary designers. Though the final form of presentation must be predicated on an analysis of the comparative effects of the variations in each argument, the authors have the following tentative form in mind:

1. Four equally-spaced values of longitudinal coefficient.
2. Four equally-spaced values of block coefficient as sub-heads under each value of longitudinal coefficient.
3. Four values of angle of inclination (15, 30, 45, and 60) as sub-heads under each value of block coefficient.
4. Curves of GZ/B versus D/H for five values of B/H (2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, & 4) for each angle of inclination.
5. Correction curves to the above values for varying values of flare, sheer, and waterline coefficient.

The above plan for presentation was decided upon after an analysis based on the usual theory and on the derived postulate set forth by Latimer and Ramsey (5) that for constant B/H and H/D the variables having the greatest effect

on righting arm are b, m, and θ. The conclusions seem logical and, therefore, result in the above tentative form, or a slight variation thereto. It is possible, however, that one of the other characteristics such as flare, sheer, or waterline coefficient might have more effect than it is now thought to have. In this event it may be necessary to have more major arguments or sub-heads in the final presentation of data.

Although completion of the entire investigation, including the analysis and presentation, will necessarily require very many man-hours of work, the authors believe that the objective sought is well worth the length of time necessary to its development. A successful attainment of the objective will enable the designer to predict conveniently and rapidly the statical stability characteristics of any proposed design. The preliminary designer will, in addition, be able to decide after only a few more minutes what changes must be made in his design in order to arrive at the desired stability characteristics.

It is recommended, then, that future investigators continue the above-mentioned procedure of collecting the necessary data and that, specifically, the below-listed steps be adhered to in the part of the investigation immediately following this one:

1. Design, by the use of Taylor's Mathematical Lines, of Hull Series "C" and "D", both to have a longitudinal coefficient equal to 0.62 and block coefficients equal to those of Hull Series "A" and "B" respectively. In both of these series, all other characteristics and coefficients to remain as in "A" and "B".#

2. Extension of series "C" and "D" to give different body plans having B/H equal to 2, 2½, 3, 3½, & 4 respectively and subsequent integration for stability of these body plans for D/H ratios of 1.4, 1.6, 1.8 & 2.0.

3. Collection and presentation of these data for Series "C" and "D" in a manner similar to that for "A" and "B" as included in this report.

4. Comparisons with stability curves of actual ships of the data obtained up to this point in the investigation. This necessarily will be a rough comparison since, for purposes of interpolation, linearity will be assumed throughout a long range of l and b; and, in addition, the effects of variation in sheer, flare, and waterline coefficient will be ignored.

5. Recommendations, based upon these comparisons, as to the values of l, b, p, flare, and sheer to be used in further hull form variations.

It will be necessary to allow the values of deadrise coefficient in Taylor's equations to change in order to arrive at fair curves for the body section. This disparity should, however, have negligible effect on stability, and cannot be obviated in any event since the deadrise coefficient is, as explained in Appendix "A", not subject to control.

It must also be remembered that the flare referred to in this report is the flare of the unit ship (i.e., the value

(Cont'd)

actually used in the computation of the mathematical hull) and not necessarily the flare on any one body plan. It is obvious that the actual flare appearing on any one body plan will be dependent upon B/H as well as upon the value of the mathematical flare on the unit ship.

S

VI APPENDIX

APPENDIX A

The Use of Taylor's Mathematical Lines

These Lines are designed to produce, not simple hulls, but actual ships, and are thus much too complete for our purposes. Moreover, the authors found that their use necessitated a good deal more knowledge than is available from a simple perusal of the text (7). Some three months were spent in their study, and in making certain necessary changes and simplifications; and to make it unnecessary for succeeding investigators to repeat these, this Appendix is included in the Thesis.

An analysis of the fairly simple, but tedious, mathematics involved will not be given, since reference (7) provides all that is needed. Briefly, however, the Lines are constructed as follows:

Knowing the values of m, b, l, and p, the designer proceeds to calculate (1) the Designer's Water Line, (2) the Curve of Sectional Areas, and (3) the Section Offsets. These calculations determine a three-dimensional, fair solid; no fairing of points is necessary, and no further adjustments need be made to the hull coefficients.

A. Designer's Water Line

Curve of Sectional Areas

These curves are of the same general shape, and the type of equations used for both is a fifth-order parabola. The constants and parameters found for one are, therefore, ap-

plicable to the other. The governing conditions for these parabolae are:

1. At the midship section, both Designer's Water Line and Sectional Area curves must be horizontal.
2. At the midship section, both Designer's Water Line and Sectional Area curves must have the ordinate equal to the desired value of beam and sectional area. If the midship section is taken as the section of greatest area and beam, as is usually the case, this will be the maximum ordinate.
3. At the midship section, the curvature must be subject to control.
4. At the bow and stern, the slope of the curves must be subject to control.
5. The coefficient of fineness of the curves (waterline coefficient, p, and longitudinal coefficient, l) must be subject to control.

In order to make these Lines of general application, Taylor (7) has used a "unit ship"; that is, one in which the half-beam, the half-length, and the draft are all set at unity. All offsets, therefore, are proper fractions, and are converted to actual offsets simply by multiplying by the actual values of B/2, L/2, or H.

Before starting the calculation of these two curves, it is necessary to determine three parameters:

1. Coefficient of fineness p or l

2. Bow tangent t

3. Acceleration (curvature) at the midship section a_1

The first of these presents no difficulty, since the designer presumably knows his longitudinal coefficient and his waterline coefficient. The determination of the other two, however, is a different matter.

Not included in reference (7), but given in certain forms used by the U. S. Model Basin*, are these Governing Equations:

$$\alpha_0 = -60 + 120\rho - \alpha_1 - 12t$$

$$S_0 = 600 - 1080\rho + 12\alpha_1 + 12t$$

$$S_1 = 240 - 360\rho + 12\alpha_1 + 12t$$

$$t = 10\rho - 5 - \frac{\alpha_1 + \alpha_0}{12}$$

and we are told that:

a_1 may be zero or negative.

a_0 is negative for full lines, positive for hollow lines.

S_0 is the slope of the acceleration curve (y"); i.e., the rate of change of curvature, at the origin.

S_1 is the slope of the acceleration curve (y") at the midship section.

---

* Obtained from Professor H. H. W. Keith of the Massachusetts Institute of Technology.

Now, suppose we decide on:

1. Full lines for both the Designer's Water Line and the Curve of Sectional Areas,
2. Maximum ordinate for both to be at the midship section,
3. Straight run,

which are reasonable. Then we must also accept:

1. a_0 negative,
2. a_1 zero,
3. S_0 zero.

From the Governing Equations, then, we get:

$$0 = 40 - 60p - \alpha_0 \qquad ----- (1)$$

$$t = 10p - 5 - \alpha_0/12 \qquad ----(2)$$

Eliminating a_0 between these two equations, we obtain:

$$25 = 45p - 3t \qquad ------ (3)$$

From equation (1), and remembering that we have said a_0 is to be zero, we see that:

$$p > 0.667$$

If we arbitrarily set p at 0.70, as was done in Hull Series "A" and "B" of this investigation, then:

$$p = 0.70$$

$$a_0 = -2.0$$

$$t = 2.17$$

Note that with the conditions we chose, we were not free to assume any values of p, a_0, and t, but were forced to take

values consistent with our assumptions and with each other. Since we have three of these parameters, and two equations involving them, we may choose only one at will, and our choice of it is determined by the type of hull we desire.

Taylor's Mathematical Lines are based on a half-ship, as explained. That is, the origin of the parabola is taken at one end of the ship, either at the bow or at the stern. At $x = 1.0$, therefore, we are at the midship section, regardless of whether we are computing the fore- or after-body. The two curves making up the complete ship must obviously fair in at this point in order to have a ship-shaped form. This means only that:

S_1 (bow) must equal S_1 (stern),

and a_1 (bow) must equal a_1 (stern).

By equating the expressions for these quantities, we may obtain:

$$\alpha_1 = 0$$

$$\alpha_{oa} = 240(P_f - P_a) + \alpha_{of} \qquad ----(4)$$

Obviously, if a_o is to be negative (for the full lines we desire), then:

$$P_a > P_f$$

which is of course reasonable. Substituting the values chosen in equation (4), and setting the after-body p at 0.84, we find that:

$$a_{oa} = -33.6$$

$$t_a = 6.2$$

$$a_{1a} = 0.0$$

As a matter of curiosity, we may find that at the stern,

$$S_{oa} = 140$$

which simply indicates a blunt stern.

The values used in the example above were those used in the calculation of Hull Series "A" and "B".

The same analysis applies exactly to the calculation of the Curve of Sectional Areas, except that in the place of waterline coefficient, p, we use the longitudinal coefficient, l. It was decided to use the longitudinal coefficient of the forebody equal to that of the after-body both because it simplifies the calculation somewhat, and because no useful purpose is served, in the majority of hulls, by varying it. This will give a Curve of Sectional Areas symmetrical about the midship section.

The shape of these curves is subject to a good deal more control than is indicated in this exposition. For example, if for a high-speed ship we want a hollow Designer's Water Line--one which has a point of inflection between the bow and the midship section--we may not only provide for it, we may control its position. The text (7), page 38, gives a series of contour curves for this purpose. These are not sufficiently accurate for our use, however, and the same analysis used above must be resorted to. Nevertheless, as explained in

the reference (7), they may be used to find the approximate position of a point of inflection, if one occurs, and to indicate general areas from which values of t corresponding to the chosen p or 1 must be selected. Thus, the values used by the authors will be found in an area where no points of inflection are possible, a necessary condition for full lines. Had hollow lines been desired, a value of t less than 2.0 is indicated. Then, with our original assumption, the use of $p = 0.70$ is not permissible, but must be some lesser value as determined by equation (3).

In general, the authors found the parameters t, a, and S very nebulous. Their actual appearance in a hull is hard to define, and their effects not set down in any reference we could find. The values finally used were selected only after much experimentation and calculation, and represent what we consider to be reasonable figures.

B. Sections

Taylor found that all ship sections cannot be handled by the same type of mathématical equation. Fine sections, for example, conform nicely to a fourth-order parabola, but full sections must be handled by a hyperbolic formula. Before starting the calculation of either type, however, it is necessary to find certain constants. This is done in the table headed "Constants for Sections", a copy of which is included at the end of this Appendix. First we define:

f Flare--the tangent of the angle between the vertical and the ship's side at the Designer's Water Line, measured on the unit ship. The flare, f, of the unit ship is related to the flare, F, of the actual ship by:

$$F = fB/2H$$

R Dead-rise Coefficient--the cotangent of the angle between the horizontal and the ship's bottom, at the keel, measured on the unit ship. The dead-rise coefficient, R, of the unit ship is related to the actual dead-rise by:

$$\text{Actual Cotangent} = RB/2H$$

(This coefficient is designated in reference (7) as "l", but since this is also the symbol for the longitudinal coefficient, the authors have changed it to "R".)

m_s Section Coefficient--the ratio of the area of a given section to the area of the circumscribing rectangle. That is,

$$m_s = A_s/B_sH_s$$

For the unit ship, B/2 and H are both unity at the midship section. At the given section, H is still unity, but $B_s/2$ is the product of the Fraction of Midship Beam, y_b, (found from the Designer's Water Line calculation) and the midship half-beam, which is 1.0; while

the area of this section, A_s, is the product of the Fraction of Midship Section Area, A, (found from the Sectional Area calculation) and the area of the midship section, which is mBH. Thus m_s reduces to:

$$m_s = Am/y_b$$

m_o Section Coefficient for zero flare. This is defined in reference (7) by:

$$m_o = (m_s - f/2)/(1 - f)$$

L This coefficient, which is unnamed, is used to find the dead-rise coefficient of the hyperbolic sections. It is not defined in the text, and is of dubious utility. It appears in:

$$R = 1/L - f(1 - L)/L$$

The quantity, R, is inherent in the method of calculation of hyperbolic sections, and is found for them simply to get a curve of R along the ship. This is so that when the shift is made from hyperbolics to parabolics, the values chosen for the latter will fair in with those inherent in the former.

These various parameters are calculated. First we inspect the values of m_s, since they determine the type of section. For values of m_s between 0.70 and 0.75, we can obtain almost identical sections with either method of calculation. For smaller values we must use fourth-power formulae, and for larger values, hyperbolic formulae. The value at which to shift from one to the other is suggested in the text as 0.72, but as a practical

matter the hyperbolic sections are so much more satisfactory that the authors used 0.70 with good results.

1. <u>Hyperbolic Sections</u>

For computing a hyperbolic section, we need only the following:

m_o

f

y_b

From the value of m_o and the curve on page 52 of reference (7), we obtain a value of the function $\emptyset(x)$ for each waterline, and simply follow down the table. With a calculating machine, a hyperbolic section may be computed in less than fifteen minutes.

2. <u>Fourth-Power Sections</u>

These present a good bit more difficulty, since we must include the Dead-rise Coefficient, R, in the calculation, and it is purely arbitrary. A curve of R for the hyperbolic sections (which should be computed first) will aid somewhat in determining its value for the fourth-power sections. In a ten-station hull, however, this is by no means infallible. The value of R is not sensitive. The authors varied it in steps of 0.5 in the indicated direction and obtained good results. Nevertheless, it was sometimes necessary to make two or three (in one case, five) calculations before obtaining a satisfactory section. No wholly satisfactory method of fixing R was ever found. The text makes no mention of any, and various tries by

the authors failed abysmally; the most direct approach is to assume a value of R as indicated by the curve mentioned above, calculate the section, plot it in relation to the hyperbolic sections already computed, and change R in the direction shown by the plot and the adjacent sections, so as to have a fair hull. (The authors found that all hulls and all their variations were best plotted on coordinate paper, both because of the time saved and the accuracy achieved.)

Of course changes in the section shape may be made by varying f, but rather than introduce one more variable into an already overcrowded field, the authors preferred to keep this parameter constant between hull forms. The values used were taken from the example ship worked out by Taylor (7), and are as follows:

Station	flare
0	0.000
1	.235
2	.240
3	.032
4	.000
5	.000
6	.000
7	.055
8	.333
9	1.500
10	.000

As stated, these Lines are too complete for the purposes of a stability investigation. The authors have, therefore, expended some effort in condensing the calculations and

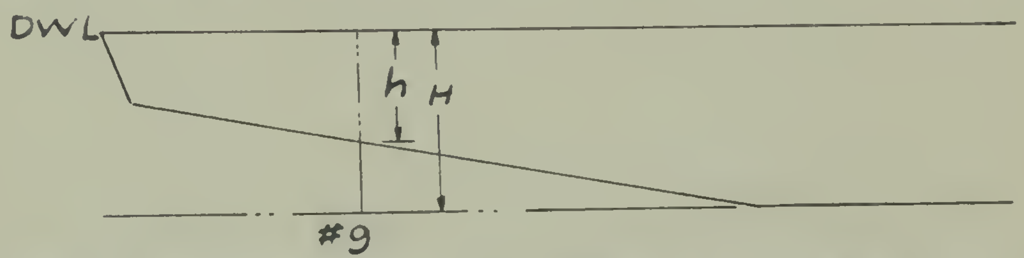
in simplifying certain portions. Briefly,

1. Ten stations were used instead of forty, the end stations having zero area.
2. The two halves of the unit ship were welded, so to speak, so that one calculation form serves to compute the entire hull.
3. Half-siding was omitted entirely.
4. Bulbous bows were not used.
5. For the sections, ten equally-spaced water-lines were used, plus a #½ WL to aid in drawing the curve. The #1.2 WL provided for in the text was not used, since the above-water body was faired in by eye.
6. Certain simplifications were made in the "Constants for Sections" Table, as explained.
7. No attempt was made to convert the Fraction of Midship Beam, y_b, or the Fraction of Midship Section Area, A, to actual offsets or areas, since by this method we retain a perfectly general hull form.

These have been assembled into a series of computation forms, samples of which are included at the end of this Appendix. Their use cuts down the time of computation immeasurably. The bold-face figures in these forms are the constants computed by Taylor; the fine figures are the computations gone through for Hull Series "B".

It is difficult to give any rigid estimate of the time required to compute an entire hull, since it seems to vary about inversely as the fineness. However, with a very full ship, one in which all the sections may be computed with the hyperbolic formulae, the calculation may be completed in less than four hours with these forms and a calculating machine. For a very fine ship, one in which all the sections must be computed by fourth-power formulae, the calculation may take up to fifteen hours, although it can be done in about seven, if the calculator is fortunate. This discrepancy is due entirely to the necessity for adjusting the values of R, plus the added complexity of the fourth-power forms.

In Taylor's Mathematical Lines as given in reference (7) and the foregoing discussion, the resulting hulls will all be of the merchant type. That is, the flat keel remains at the base line for the entire length of the ship. Naval vessels are not normally built in this fashion, but have the dead-wood cut away severely aft of about Station #7:



The Lines as given by Taylor make no provision for this idiosyncrasy. If such a profile is desired, however, the authors have evolved a very simple method for obtaining it.

First, draw the profile. Measure carefully the distance from the base line to the bottom of the flat keel, and

the draft to the Designer's Water Line at each station affected. This draft is denoted as h. Then the section coefficient of this station is:

$$m_s' = A_s/B_s h$$

The section coefficient of the station obtained from the Mathematical Lines will be:

$$m_s = A_s/B_s H$$

We naturally desire the same sectional area for each of these hulls, since to have otherwise would change the longitudinal coefficient of the ship. We equate the expressions for A_s, and find that:

$$m_s' = m_s \times H/h$$

If we substitute this new value of section coefficient in the computation forms and complete the calculation, we will have the offsets of the desired station. The waterlines will be spaced closer together in the ratio of h/H, of course, but all we have to do is to start laying off the offsets at the new draft, h, using the new waterlines. Where the original waterlines cross the outline of the new section, we can pick off the offsets to go with the other sections, if their numerical value is necessary to some calculation. For the purpose of stability, this will not be the case, and the outline itself is all we will need. Incidentally, this will probably simplify the hull computation a little, since the new value of m_s' will always be larger than the original m_s, and may well be large enough to allow a hyperbolic section to be used rather than a fourth-power section.

There is one drawback to this, from the authors' point of view. Recalling the formula for m_o:

$$m_o = (m_s - f/2)/(1 - f)$$

we see that if f is greater than 1.0, m_o turns out to be negative. However, it is also true that in the stations where such a condition would be present (Station #9 in Hull Series "A" and "B" was in this category), the value of m_s is small enough so that the numerator of the fraction is also negative, and m_o is positive. If, however, the value of m_s is increased in the ratio of H/h, the numerator may well turn out to be positive, and then m_o is negative. For the use of Taylor's Mathematical Lines, this is not permissible, since the curves of $\emptyset(x)$ versus m_o are not defined below about 0.60. We have no choice, then, but to change the value of flare, f, sufficiently to give us a usable value of m_o. The authors do not believe that this would be a fatal error in this stability calculation, nor do they feel that a hull profile of this sort will introduce any major changes in the overall stability characteristics of a ship, largely because the stations affected are at the ends of the ship and are of small area. Nevertheless, the order of magnitude of the change would be a matter of useful knowledge.

HULL SERIES: B

$b = .469$
$m = .70$
$\ell = .67$
$p = .77$

CONSTANTS FOR SECTIONS

STA.	A	A·m	y_B	m_s	f	$\frac{f}{2}$	$m_s - \frac{f}{2}$	1−f	m_o	L	(1−L)	R
1	.3078	.2155	.3936	.5475	.235	.1175	.430	.765	.562	These quantities not		
2	.6526	.4568	.6946	.6576	−.40	.120	.5376	.760	.707	necessary for fourth		
3	.8864	.6205	.8924	.6456	.032	.016	.6636	.968	.686	power sections.		
4	.9803	.6847	.9841	.7008	0	0	.7008	1.0	.701			
5	1.0000	.7000	1.00	.7000	0	0	.7000	1.0	.700			
6	.9853	.6897	.9909	.6960	0	0	.6960	1.0	.696			
7	.8869	.6208	.9621	.6452	.055	.0275	.6177	.945	.654			
8	.6526	.4568	.9153	.4991	.333	.1665	.3326	.667	.499			
9	.3078	.2155			.150	.750	This station faired in by hand to have correct area					

$$m_s = \frac{A \cdot m}{y_B} \qquad m_o = \frac{m_s - \frac{f}{2}}{1-f} \qquad R = \frac{1-f(1-L)}{L}$$

HULL SERIES: B

$b =$ 467

$m =$ .7

$\ell =$ 67

$p =$ .31

FOURTH-POWER SECTIONS

STA* 1 $m = .5410$ $f = 0.255$ $R = 2.34$ $y_B = .3936$

x	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.2
M	.06769	.2430	.7680	1.323	1.728	1.875	1.728	1.323	.7680	.2430	0	1.728
F	.00327	.01125	.03200	.04725	.04800	.03125	0	-.03675	-.06400	-.06075	0	.4320
L	.03948	.06075	.06400	.03675	0	-.03125	-.04800	-.04725	-.03200	-.01125	0	-.09600
Y	-.0266	-.0935	-.2800	-.4455	-.5120	-.4375	-.2160	.1225	.5120	.8505	1.0	0
m·M	.0371	.1330	.4205	.7243	.9461	1.0266	0.9461	.7243	.4205	.1330	0	.3746
f·F	.0008	.0026	.0075	.0111	.0113	.0073	0	-.0086	-.0150	-.0143	0	.1015
R·L	.0942	.1421	.1498	.0860	—	-.0731	-.1123	-.1106	-.0749	-.0263	0	-.2246
y_s	.1055	.1836	.2978	.3721	.4454	.5233	.6178	.7276	.8426	.9429	1.0	not used
$\bar{Y}$	.0415	.0723	.1172	.1485	.1753	.2060	.2432	.2864	.3316	.3711	.3436	!

HULL SERIES ___

$b =$ ___
$m =$ ___
$\ell =$ ___
$p =$ ___

HYPERBOLIC SECTIONS

STA # ___	$f =$	$(1-f) =$	$m_o =$	$y_B =$								
x	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.2
$\varphi(x)$												
$(1-f)\cdot\varphi(x)$												
$f\cdot x$												
y_s												
Y												

HULL SERIES B

COEFFICIENTS:

$b =$ .463

$\ell =$ .670

$m =$ 10

$p =$ 17

PARAMETERS:

	FWD.	AFT.
$t =$	2.17	6.2
$\alpha_1 =$	0	0
$p =$	20	84

DESIGNER'S W.L.

STA.	1	2	3	4	5	6	7	8	9
C_t	.04096	-.01728	-.03072	-.00896	0	-.00896	-.03072	-.01728	.04096
C_α	-.00768	-.00576	.00576	.00768	0	.00768	.00576	-.00576	-.00768
C_p	1.2288	2.0736	1.3824	0.3072	0	0.3072	1.3824	2.0736	1.2288
C_y	-.5565	-.7194	-.0086	0.7885	1.0	0.7885	-.0086	-.7194	-.5565
$t\,C_t$	.0889	-.0375	-.0667	-.0194	—	-.0556	-.1905	-.1071	.2540
$\alpha_1 \cdot C_\alpha$	—	—	—	—	—	—	—	—	—
$p \cdot C_p$	.8602	1.4515	.4677	.2150	—	.2580	1.1612	1.7418	1.0322
y_B	.3926	.6946	.8924	.9941	1.0	.4409	.4612	.9153	.1297

	FWD	AFT
$t =$	1.0	1.0
$\alpha_1 =$	0	0
$\ell =$	.067	.067

SECTIONAL AREAS

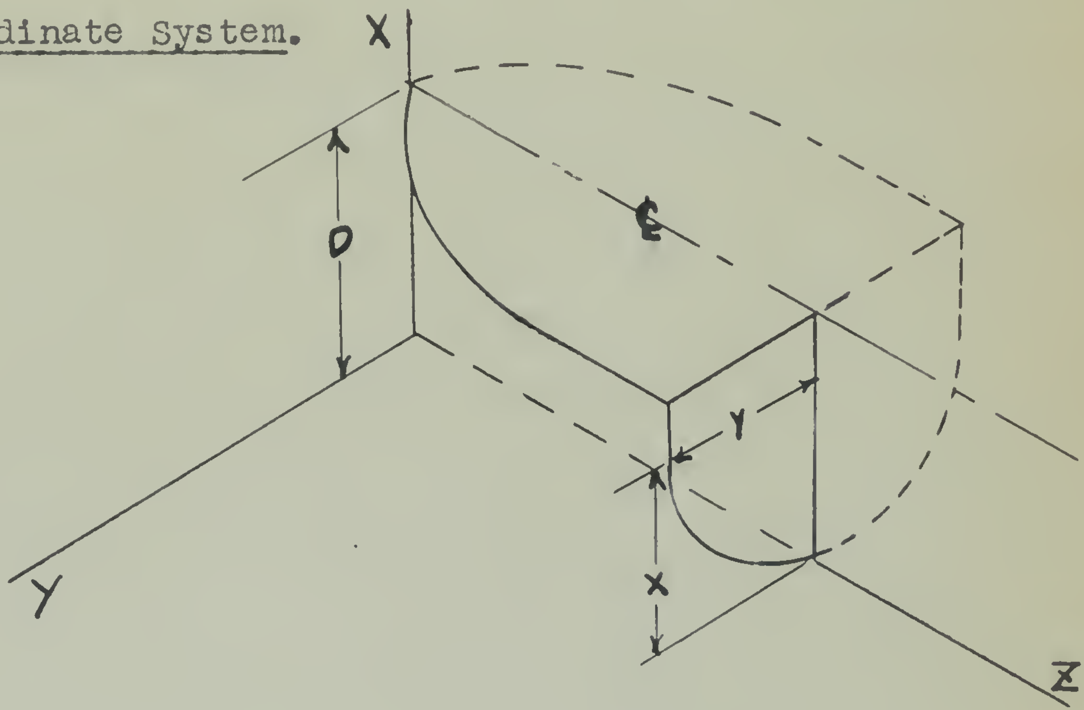
STA.	1	2	3	4	5	6	7	8	9
C_t									
C_α									
C_p									
C_y									
$t\,C_t$	.0410	-.0173	-.0307	.0090	—	-.0090	-.0307	-.0173	.0410
$\alpha_1 C_\alpha$	—	—	—	—	—	—	—	—	—
$\ell\,C_p$	.8233	1.3893	.9262	.2058	—	.2058	.9262	1.3893	.8233
A	.3078	.6526	.8869	.9853	1.0	.9853	.8869	.6526	.3078

APPENDIX B

A Mathematical Analysis of Statical Stability

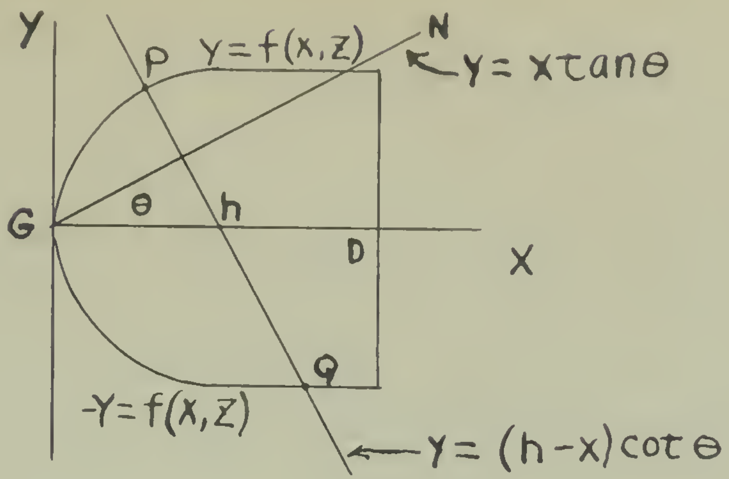
Mathematical lines defining the ship form suggest the possibility of developing mathematical equations for statical stability. It is the object of this appendix to develop general equations for immersed volume and righting moment using the hypothesis that analytical mathematical lines are available, and then to show the difficulties in adopting Taylor's Mathematical Lines to the general equations.

General Coordinate System.



Origin of coordinates at forefoot,
Z axis along keel,
xz plane is C.L. plane,
yx planes are section curves.

Considering any transverse section, for the moment, (z fixed):



ϕ - angle of inclination.

h - intersection of inclined water plane and centerline plane.

P - X coordinate of shallow intersection of water plane and shell.

Q - X coordinate of deep intersection of water plane and shell.

D - depth.

V - immersed volume (L^3).

M - righting moment (L^4).

Assuming G at the keel for convenience, we take the first moment of the immersed sectional area about an axis through the origin and perpendicular to the water plane.

$$\frac{dV}{dz} = 2\int_0^P y\,dx + \int_P^Q \left[y + (h-x)\cot\theta \right] dx \qquad [1]$$

$$\frac{dM}{dz} = 2\sin\theta\int_0^P yx\,dx + \cos\theta\int_P^Q \left[y + (h-x)\cot\theta \right]\left[y + - \frac{1}{2}(h-x)\cot\theta - \frac{y}{2} + x\tan\theta \right] dx$$

$$= 2\sin\theta\int_0^P yx\,dx + \cos\theta\int_P^Q \left[y^2 + xy\tan\theta + x(h-x) - \frac{(h-x)^2\cot^2\theta}{2} \right] dx$$

$$[2]$$

Now integrating along the length of the ship:

$$V = \int_0^L [1]\, dz \qquad ; \qquad M = \int_0^L [2]\, dz$$

$$V = 2\int_0^L\int_0^P y\, dx\, dz + \int_0^L\int_P^Q [y + (h-x)\cot\theta]\, dx\, dz \qquad [3]$$

$$M = 2\sin\theta\int_0^L\int_0^P xy\, dx\, dz + \cos\theta\int_0^L\int_P^Q \left[\frac{y^2}{2} + xy\tan\theta + x(h-x) - \frac{(h-x)^2\cot^2\theta}{2}\right] \qquad [4]$$

For any given angle of inclination, ϕ, and inclined draft, h, solution of equations (3) and (4) would give one point on a cross curve of statical stability.

The limits of integration, P and Q, would have to be obtained from the equation for the intersection of the water plane and the shell, as follows:

$$y = f(x, z) = (h-x)\cot\theta \; ; \; [x = P] \qquad [5]$$

$$-y = f(x, z) = (h-x)\cot\theta \; ; \; [x = Q] \qquad [6]$$

When the deck edge is immersed, note that Q remains equal to D. Hence, equation (6) no longer applies. The angle of deck edge immersion is found from the relation:

$$\tan\theta = \frac{h - D}{f(x, z)} \qquad [7]$$

It is readily seen that for a given ϕ and h, P and Q are functions of z.

The general equations (3) and (4) are useful only if: (1) the ship solid can be defined in such a manner that y is analytical in x and z; and, (2) the equations for determining P and Q, (5 and 6), can be solved in terms of z, ϕ, and h.

Unfortunately, these requirements are not fulfilled by Taylor's Mathematical Lines. Hence, it is necessary to modify equations (3) and (4) to lend themselves to numerical integration for such lines.

Taylor's sectional area curve for fine sections is given in the form:

$$y = Y + Mm + Ff + LR$$

where Y, M, F, and L are functions of x alone and could be easily integrated in equations (3) and (4). Also, unit section coefficient (m) is analytical by means of Taylor's equation for the curve of sectional areas. However, flare (f) and deadrise coefficient (R) are not easily definable as functions of z, as explained in Appendix (A).

A similar problem arises in Taylor's formula for full sections:

$$y = fx + (1-f)(1+c^2)\left[1 - \frac{cx}{(1+c)^2} - \frac{c}{x+c}\right]$$

in which f and c are not readily definable as functions of z.

The possibility of numerical integration of equations (3) and (4) is now considered.

Taylor's coefficients (R, f, m, etc.) are based upon unit curves such that x is one when y is one. If we let $x = \frac{X}{D}$ and $y = \frac{2Y}{By_b}$, where X and Y are the general coordinates of equations (3) and (4), and y_b is a fraction such that By_b is the deck width of the section in question, then equations (3) and (4) become:

$$V = \frac{LD}{10} \sum_{sta.\,0}^{sta.\,10} By_b \int_0^P y\,dx + \int_P^Q \left[\frac{By_b y}{2} + D(h-x)\cot\theta\right] dx \qquad [3']$$

$$M = \frac{LD}{10}\sum_0^{10}\left\{DBy_b \sin\theta\int_0^P xy\,dx + \cos\theta\int_P^Q\left[\frac{(By_by)^2}{8} + \frac{DBy_b\,xy\tan\theta}{2} + D^2(h-x)x - \frac{D^2(h-x)^2\cot^2\theta}{2}\right]dx\right\} \qquad [4']$$

Note that x and y are now dimensionless coordinates of unit curves such that x is one when y is one.

By replacing B by $\frac{B}{D}$ x D and simplifying, we obtain:

$$V = \frac{LD^2}{10}\sum\left\{\frac{B}{D}y_b\int_0^P y\,dx + \int_P^Q\left[\frac{B}{2D}y_by + (h-x)\cot\theta\right]dx\right\} \qquad [3'']$$

$$M = \frac{LD^3}{10}\sum\left\{\frac{B}{D}y_b\sin\theta\int_0^P xy\,dx + \cos\theta\int_P^Q\left[\frac{B^2y_b^2y^2}{D^2 8} + \frac{B}{2D}y_bxy\tan\theta + (h-x)x - \frac{(h-x)^2\cot^2\theta}{2}\right]dx\right\} \qquad [4'']$$

Since $\frac{GZ}{D} = \frac{M}{VD}$, we obtain the following dimensionless expression for righting arm as a fraction of depth:

$$\frac{GZ}{D} = \frac{\sum_0^{10}\frac{B}{D}y_b\sin\theta\int_0^P yx\,dx + \cos\theta\int_P^Q\left[\frac{B^2y_b^2y^2}{8D^2} + \frac{Bx}{2D}y_by\tan\theta + (h-x)x - \frac{(h-x)^2\cot^2\theta}{2}\right]dx}{\sum_0^{10}\frac{B}{D}y_b\int_0^P y\,dx + \int_P^Q\left[\frac{B}{2D}y_by + (h-x)\cot\theta\right]dx} \qquad [8]$$

Equation (8) can be solved by numerical integration of Taylor's Lines substituting for y the equation for each section after m, f, and R have been determined. Also, it is necessary to solve the intersection equations (5, 6, and 7) for each section considered.

Equation (8) shows several interesting facts about the stability of any mathematical hull form.

(a) The dimensionless righting arm, $\frac{GZ}{D}$, is independent of length, (L).

(b) $\frac{GZ}{D}$ is a function of $\frac{B}{D}$, Ø, h, and the form function y.

(c) For a given ship with $\frac{B}{D}$ and y fixed, $\frac{GZ}{D}$ is a function of Ø and h alone.

Equation (8) is a perfectly general equation for righting arm and does not depend on the usual assumptions that sheer is zero or that longitudinal trim during inclination is constant, since depth (D) and inclined draft (h) may be considered as functions of z. To find the longitudinal trim after inclination so that the correct h (h is a function of z) may be substituted in equation (8) for the stability corrected for longitudinal trim, use can be made of the following equation based on the restraint that the longitudinal moment after inclination must be zero:

$$\int_0^L \frac{dV}{dz} (z - G_z)\, dz = 0 \qquad [9]$$

V is a function of h, hence solution of equation (9) would give h as a function of z. This value of h should be substituted in equation (8) if correction for longitudinal trim is desired.

The analytical methods outlined in this appendix were rejected by the authors insofar as their specific application to Taylor's Mathematical Lines is concerned because the process of actually drawing body plans from offsets calculated by Taylor's method and analyzing these for stability by mechanical integration seemed to be a more positive and rapid means of reaching the objective of the thesis.

Nevertheless, it is felt that future investigators may find an extension of the analytical methods outlined here to be fruitful. Work similar to that of Latimer and Ramsey

(see ref. 5), where simple geometric forms are investigated for statical stability, could be done readily by equations (3) and (4) for forms analytical in z and equations (3') and (4') for forms not analytical in z. This would have the advantage over the integrator method that longitudinal trim during inclination need not be ignored. Also, it might prove to be of value to differentiate equation (8) and set it equal to zero to obtain an expression for the maximum righting arm for various form functions (y). In like manner, an expression for range of stability may be obtained by equating equation (8) to zero.

Example of the Use of Equation (8).

Consider a square ended barge. Then $B/D = 1$, $y_b = 1$, and the form function $y = 1$.

Equation (8) simplifies to:

$$\frac{GZ}{D} = \frac{\sin\theta \int_0^P x\,dx + \cos\theta \int_P^Q \left[\frac{1}{8} + x\frac{\tan\theta}{2} + hx - x^2 - \frac{(h^2 - 2hx + x^2)\cot^2\theta}{2}\right]dx}{\int_0^P dx + \int_P^Q \left[\frac{1}{2} + h\cot\theta - x\cot\theta\right]dx}$$

$$= \frac{\sin\theta\; P^2/2 + \cos\theta\left[\frac{Q-P}{8} + \frac{(Q^2-P^2)}{4}(\tan\theta + 2h) - \frac{(Q^3-P^3)}{3} - \frac{\left(h^2(Q-P) - h(Q^2-P^2) + (Q^3-P^3)\right.}{2}\right.}{P + \frac{Q-P}{2}(1 + 2h\cot\theta) - \frac{(Q^2-P^2)}{2}\cot\theta}$$

FOR THE CASE WHERE $\theta = 45°$ AND $h = 1/2$:

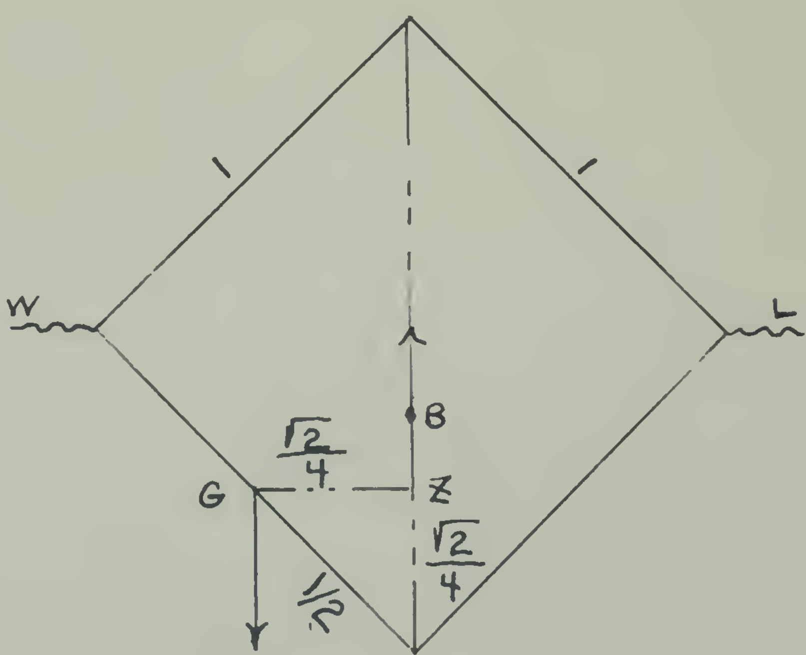
$$P = h - \frac{\tan\theta}{2} = 1/2 - 1/2 = 0$$

$$Q = h + \frac{\tan\theta}{2} = 1/2 + 1/2 = 1$$

$$\frac{GZ}{D} = \frac{0 + \frac{1}{\sqrt{2}}\left[\frac{1}{8} + \frac{1}{2} - \frac{1}{3} - \frac{1}{2}\left(\frac{1}{4} - \frac{1}{2} + \frac{1}{3}\right)\right]}{0 + \frac{1}{2}(2) - \frac{1}{2}}$$

$$= \frac{\sqrt{2}}{4} \quad \text{ANSWER}$$

CHECK:



APPENDIX C

Summary of Data and Calculations

I. Hull Series "A":

Hull Coefficients:	DWL Parameters: Fwd.	Aft.	Sectional Area Parameters: Fwd.	Aft.
l = 0.67	t = 2.17	6.2	t = 1.0	1.0
b = 0.608	a_1 = 0.0	0.0	a_1 = 0.0	0.0
p = 0.77	p = 0.70	0.84	l = 0.67	0.67

Sta.	1	2	3	4	5	6	7	8	9
Flare	.235	.240	.032	.000	.000	.000	.055	.333	1.500

No camber, sheer, or half-siding.

TABLES OF GZ/B:

B/H = 2.0 GM/B = -0.052

D/H	2.0	1.8	1.6	1.4
15°	-.0135	-.0135	-.0135	-.0135
30°	-.0152	-.0152	-.0152	-.0188
45°	.0065	.0038	-.0110	-.0386
60°	.0378	.0126	-.0105	-.0665

B/H = 2.5 GM/B = 0.0331

D/H	2.0	1.8	1.6	1.4
15°	.0110	.0110	.0110	.0110
30°	.0293	.0293	.0279	.0135
45°	.0648	.0532	.0317	-.0013
60°	.0777	.0487	.0126	-.0204

B/H = 3.0 GM/B = 0.104

D/H	2.0	1.8	1.6	1.4
15°	.0268	.0268	.0268	.0268
30°	.0644	.0644	.0562	.0342
45°	.0954	.0776	.0494	.0150
60°	.0872	.0640	.0276	-.0154

B/H = 3.5 GM/B = 0.167

D/H	2.0	1.8	1.6	1.4
15°	.0442	.0442	.0442	.0442
30°	.0966	.0944	.0812	.0564
45°	.1200	.1015	.0739	.0335
60°	.1080	.0783	.0422	.0044

B/H = 4.0 GM/B = 0.2245

D/H	2.0	1.8	1.6	1.4
15°	.0618	.0618	.0618	.0618
30°	.1200	.1115	.0974	.0660
45°	.1367	.1123	.0858	.0429
60°	.1136	.0846	.0515	.0169

II. Hull Series "B":

Hull Coefficients:	DWL Parameters:	Sectional Area Parameters:
l = 0.67 b = 0.469 p = 0.77	As in Series "A".	As in Series "A".

Flare as in Series "A".

No camber, sheer, or half-siding.

TABLES OF GZ/B:

B/H = 2.0 GM/B = 0.0285

D/H	2.0	1.8	1.6	1.4
15°	.0089	.0089	.0089	.0089
30°	.0208	.0208	.0208	.0158
45°	.0449	.0442	.0316	.0014
60°	.0816	.0565	.0206	-.0142

B/H = 2.5 GM/B = 0.123

D/H	2.0	1.8	1.6	1.4
15°	.0304	.0304	.0304	.0304
30°	.0614	.0614	.0607	.0446
45°	.0911	.0836	.0610	.0263
60°	.1101	.0806	.0434	-.0024

B/H = 3.0 GM/B = 0.201

D/H	2.0	1.8	1.6	1.4
15°	.0488	.0488	.0488	.0488
30°	.0875	.0875	.0834	.0620
45°	.1210	.1060	.0811	.0450
60°	.1270	.0951	.0565	.0129

B/H = 3.5 GM/B = 0.273

D/H	2.0	1.8	1.6	1.4
15°	.0665	.0665	.0665	.0665
30°	.1167	.1167	.1035	.0765
45°	.1430	.1230	.0951	.0555
60°	.1370	.1040	.0675	.0245

B/H = 4.0 GM/B = 0.339

D/H	2.0	1.8	1.6	1.4
15°	.0843	.0843	.0843	.0820
30°	.1410	.1360	.1200	.0915
45°	.1595	.1365	.1080	.0685
60°	.1450	.1120	.0794	.0370

APPENDIX D

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